Design of Flexural Members

Laterally supported and un-supported beams

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Design of Beams

A beam is a structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight and external reactions to these loads is called a bending moment. The beam is also subjected to shear force. Beams generally carry vertical gravitational forces but can also be used to carry horizontal loads (i.e., loads due to an earthquake or wind). The loads carried by a beam are transferred to columns, walls, or girders, which then transfer the force to adjacent structural compression members. In Light frame construction the joists rest on the beam.

Beams are characterized by their profile (the shape of their cross-section such as I, H, O etc.,), their length (l), and their material (E). In contemporary construction, beams are typically made of steel, reinforced concrete, or wood or composite materials. One of the most common types of steel beam is the I-beam (ISMB, ISLB) or wide-flange beam (ISWB also known as a "universal beam" or, for stouter sections, a "universal column"). This is commonly used in steel-frame buildings and bridges. Other common beam profiles are the C-channel ([ ), the hollow structural section beam, the pipe, the T and the angle. Compound sections and plated sections are also used for long span and for medium to heavy loads.

Structural characteristics

Internally, beams experience compressive, tensile and shear stresses as a result of the loads applied to them. Typically, under gravity loads, the original length of the beam is slightly reduced to enclose a smaller radius arc at the top of the beam, resulting in compression, while the same original beam length at the bottom of the beam is slightly stretched to enclose a larger radius arc, and so is under tension. The same original length of the middle of the beam, generally halfway between the top and bottom, is the same as the radial arc of bending, and so it is under neither compression nor tension, and defines the neutral axis. Above the supports, the beam is exposed to shear stress.

General shapes

Most beams in reinforced concrete buildings have rectangular cross sections, but the most efficient cross section is a universal beam (I section). The fact that most of the material is placed away from the neutral axis (axis of symmetry in case of universal beam) increases the second moment of area of the beam which in turn increases the stiffness. A universal beam is only the most efficient shape in one direction of bending: up and down looking at the profile as an I. If the beam is bent side to side , it functions as an H where it is less efficient. The most efficient shape for both directions in 2D is a box (a square shell) however the most efficient shape for bending in any direction is a cylindrical shell or tube. But, for unidirectional bending, the universal (I or wide flange) beam is king. Efficiency means that for the same cross sectional area (Volume of beam per length) subjected to the same loading conditions, the beam deflects less. Other shapes, like L (angles), C (Channels) or tubes, are also used in construction when there are special requirements (purlins in trusses and for tower members).

Laterally supported and un supported beams

Generally, a beam resists transverse loads by bending action. In a typical building frame, main beams are employed to span between adjacent columns; secondary beams when used – transmit the floor loading on to the main beams. In general, it is necessary to consider only the bending effects in such cases, any torsional loading effects being relatively insignificant. The main forms of response to uni-axial bending of beams are listed in Table 1.

Under increasing transverse loads, beams of category 1 [Table1] would attain their full plastic moment capacity. This type of behaviour has been covered in an earlier chapter. Two important assumptions have been made therein to achieve this ideal beam behaviour. They are:

- The compression flange of the beam is restrained from moving laterally, and
- Any form of local buckling is prevented.

If the laterally unrestrained length of the compression flange of the beam is relatively long as in category 2 of Table 1, then a phenomenon, known as *lateral buckling* or

lateral torsional buckling of the beam may take place. The beam would fail well before it could attain its full moment capacity. This phenomenon has a close similarity to the Euler buckling of columns, triggering collapse before attaining its squash load (full compressive yield load).

Lateral buckling of beams has to be accounted for at all stages of construction, to eliminate the possibility of premature collapse of the structure or component. For example, in the construction of steel-concrete composite buildings, steel beams are designed to attain their full moment capacity based on the assumption that the flooring would provide the necessary lateral restraint to the beams. However, during the erection stage of the structure, beams may not receive as much lateral support from the floors as they get after the concrete hardens. Hence, at this stage, they are prone to lateral buckling, which has to be consciously prevented. Beams of category 3 and 4 given in Table 1 fail by local buckling, which should be prevented by adequate design measures, in order to achieve their capacities. The method of accounting for the effects of local buckling on bending strength is available in IS: 800.

The conceptual behaviour of laterally unrestrained beams is described in detail in code. Various factors that influence the lateral buckling behaviour of a beam are explained. The design procedure for laterally unrestrained beams is also included.

Section classification, Design moment capacity, Section modulus (elastic and plastic), shear design provisions, imperfection factors, web buckling, web crippling, deflection, requirements, etc.: Refer IS 800.

Influence of cross sectional shape on lateral torsional buckling

Structural sections are generally made up of either open or closed sections. Examples of open and closed sections are shown in figure.

| Wide Flange Beam | Standard beam | Channel | Angle | Tee |
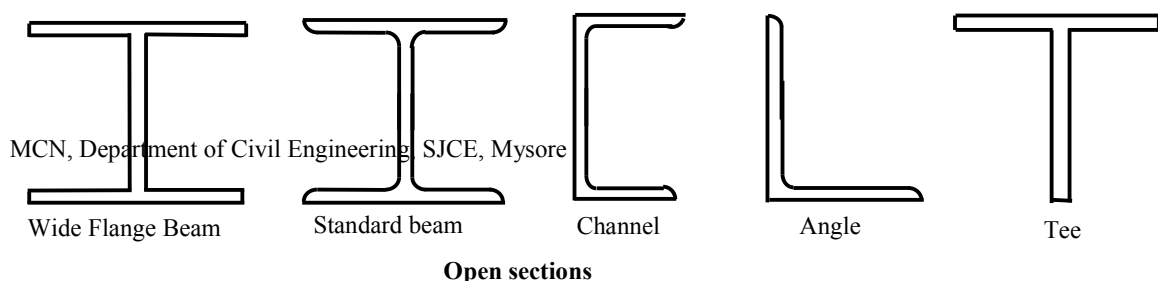
Open sections

Cross sections, employed for columns and beams (I and channel), are usually open sections in which material is distributed in the flanges, i.e. away from their centroids, to improve their resistance to in-plane bending stresses. Open sections are also convenient to connect beams to adjacent members. In the ideal case, where the beams are restrained laterally, their bending strength about the major axis forms the principal design consideration. Though they possess high major axis bending strength, they are relatively weak in their minor axis bending and twisting.

The use of open sections implies the acceptance of low torsional resistance inherent in them. No doubt, the high bending stiffness (EI_x) available in the vertical plane would result in low deflection under vertical loads. However, if the beam is loaded laterally, the deflections (which are governed by the lower EI_y rather than the higher EI_x) will be very much higher. From a conceptual point of view, the beam has to be regarded as an element having an enhanced tendency to fall over on its weak axis.

Factors affecting lateral stability

The elastic critical moment, M_{cr}, as obtained in the previous Section, is applicable only to a beam of I section which is simply supported and subjected to end moments. This case is considered as the basic case for future discussion. In practical situations, support conditions, beam cross section, loading etc. vary from the basic case. Refer IS code for details.

Table 1 Main failure modes of hot-rolled beams

Cate-gory	Mode	Figure	Comments
1	Excessive bending triggering collapse		This is the basic failure mode provided (1) the beam is prevented from buckling laterally,(2) the component elements are at least compact, so that they do not buckle locally. Such "stocky" beams will collapse by plastic hinge formation.
2	Lateral torsional buckling of long beams which are not suitably braced in the lateral direction.(i.e. "un restrained" beams)		Failure occurs by a combination of lateral deflection and twist. The proportions of the beam, support conditions and the way the load is applied are all factors, which affect failure by lateral torsional buckling.
3	Failure by local buckling of a flange in compression or web due to shear or web under compression due to concentrated loads	Refer IS:800-2007	Unlikely for hot rolled sections, which are generally stocky. Fabricated box sections may require flange stiffening to prevent premature collapse. Web stiffening may be required for plate girders to prevent shear buckling. Load bearing stiffeners are sometimes needed under point loads to resist web buckling.
4	Local failure by (1) shear yield of web (2) local crushing of web (3) buckling of thin flanges.		Shear yield can only occur in very short spans and suitable web stiffeners will have to be designed. Local crushing is possible when concentrated loads act on unstiffened thin webs. Suitable stiffeners can be designed. This is a problem only when very wide flanges are employed. Welding of additional flange plates will reduce the plate b / t ratio and thus flange buckling failure can be avoided.

Design of steel beam (ASD, Allowable Stress design)-IS:800-1984

Design requirements

1. Maximum bending stress, f_b must not exceed allowable stress, F_b.
2. Deflection should not exceed allowable limit.
3. Maximum shear stress, f_V shall not exceed allowable shear stress.

Design procedure:

1. Calculate design load.
2. Calculate design moment, M and bending stress, f_b.
3. Select a trial beam size and calculate allowable bending stress, F_b (see problems)
4. Calculate deflection and check with allowable deflection ratio.
5. Calculate design shear and shear stress, f_v.
6. Calculate allowable shear stress, F_v.

Limit state design

Design of laterally supported beams

1. Calculate the factored load and the maximum bending moment and shear force
2. Obtain the plastic section modulus required

$$Z_{req} = \frac{(M \times \gamma_{mo})}{fy}$$

   Select a suitable section for the beam-ISLB, ISMB, ISWB or suitable built up sections (doubly symmetric only). (Doubly symmetric, singly symmetric and asymmetric- procedures are different)

3. Check for section classification such as plastic, compact, semi-compact or slender. Most of the sections are either plastic or compact. Flange and web criteria.

$$\frac{d}{t_w} \ , \ \frac{b}{t_f}, \ \varepsilon = \sqrt{\frac{250}{fy}} = 1$$

4. Calculate the design shear for the web and is given by

$$V_{dp} = \frac{(Av \times fy)}{\sqrt{3} \times \gamma_{mo}} > V_d \ and, V < 0.6V_d$$

5. Calculate the design bending moment or moment resisted by the section (for plastic and compact)

   $M_d = \beta_p \times Z_p \times f_y / \gamma_{mo}$

6. Check for buckling
7. Check for crippling or bearing
8. Check for deflection

Design of laterally un-supported beams

1. Calculate the factored load and the maximum bending moment and shear force
2. Design of LSB is by trial and error method. The design bending stress is significantly less which is to be assumed to start with. Assume slenderness ratio and h/t_f and get the corresponding critical bending stress and hence the corresponding design bending stress.
3. Determine the required plastic section modulus and select the section.
4. Determine the actual design bending stress of this selected section knowing its slenderness ratio which should be greater than that assumed previously. Otherwise revise the section.
5. Check for shear, buckling, crippling and deflection should be done

Design bending strength can be calculated as per IS: 800

$$M_d = \beta_p \times Z_p \times f_{bd}$$

$$\lambda_{LT} = \sqrt{\frac{(\beta_b \times Z_{pz} \times fy)}{M_{cr}}} \quad < \text{or} = [1.2 Z_e \, f_y / M_{cr}]^{0.5}$$

$$= [\, f_y / f_{cr,b}]^{0.5}$$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\lambda_{LT} - 0.2) + \lambda_{LT}^2 \right]$$

$$X_{LT} = \frac{1}{\left[\phi_{LT} + \left[\phi_{LT}^2 - \lambda_{LT}^2 \right]^{0.5} \right]} \leq 1.0$$

α_{LT} = 0.21 or 0.49 rolled or welded sections

M_{cr} and $f_{cr,b}$ can b Elastic lateral buckling, $M_{cr} = \sqrt{\left[\left(\frac{\pi^2 E I_y}{(L_{LT})^2} \right) \left(G I_t + \frac{\pi^2 E I_w}{(L_{LT})^2} \right) \right]}$

Torsional constant, $I_t = \sum b_i t_i^3 / 3$

Warping constant, $I_w = (1 - \beta_f) \beta_f I_y h_f^2$

$\beta_f = 0.50$, $h_f = (D\text{-}t_f)$, $G = \dfrac{E}{2(1+\mu)}$

Design bending compressive stress, $f_{bd} = \dfrac{\chi_{LT} f_y}{\gamma_{mo}}$, with this M_d can be found

Analysis of section:

To find the design moment of the section.

Laterally supported beam

Determine the moment of resistance of the beam from its plastic section modulus.
Knowing the type of beam and its span, the ultimate load can be determined. Find the safe load using suitable partial safety factor for the loads.

Laterally supported beam

For the given beam determine the design moment or MOR as follows.

First find the critical moment of the section knowing its properties. Calculate the non dimensional factors and hence f_{bd}. From this M_d can be found. Problem can also be solved by using tables of IS code.

Workout problems- tutorials

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