Design of Flexural Members

CV610 – Design of steel structures

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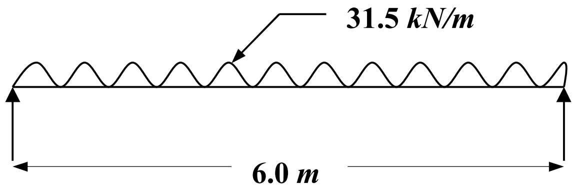
Problems on beams

LATERALLY SUPPORTED BEAMS

1. Design a simply supported beam to carry a uniformly distributed load of 20 *kN/m*. the effective span of beam is 6.0m



SOLUTION

Limit State Method

Effective span, $l_{eff} = 6.0\ m$

Assuming the self weight of beam = 1.0 *kN/m*

Total load on beam = 20.0+1.0 = 21.0 *kN/m*

Factored load = 21x1.5 = 31.5 *kN/m*

Maximum Bending moment, $M = \frac{wl^2}{8} = \frac{31.5x6^2}{8} = 141.75\ kN\text{-}m$

Maximum Shear force, $V = \frac{wl}{2} = \frac{31.5x6}{2} = 94.5\ kN$

Plastic Section modulus required

$Z_p = \frac{M\gamma_{m0}}{f_y} = \frac{141.75x10^6 x1.10}{250} = 623.70x10^3\ mm^3$

Consider section ISMB 300 @ 44.2 *kg/m*

Sectional properties:

Sectional area, $A = 5626\ mm^2$

Depth of section, $D = 300\ mm$

Width of flange, $b_f = 140\ mm$

Moment of inertia, $I_{zz} = 8603.6\ cm^4$

Moment of inertia, $I_{yy} = 453.9\ cm^4$

Plastic modulus, $Z_{pz} = 651.74 cm^3$

Radius at root, $r_1 = 14.0\ mm$

$d_2 = 29.25\ mm$

Thickness of flange, $t_f = 12.4\ mm$

Thickness of web, $t_w = 7.5\ mm$

Radius of gyration, $r_{zz} = 12.37\ cm$

Radius of gyration, $r_{yy} = 2.84\ cm$

Section modulus, $Z_{ez} = 573.6 cm^3$

Section modulus, $Z_{ey} = 64.8\ cm^3$

Depth of web, $d = 241.5\ mm$

Section classification *Table-2 of code IS: 800-2007*

$$\varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Outstanding flanges:

$$\frac{b}{t_f} = \frac{140}{2x12.4} = 5.64 < 9.4\varepsilon$$

Hence flange is plastic element.

Web of an I-section: ('d' is obtained directly from steel table)

$$\frac{d}{t_w} = \frac{241.5}{7.5} = 32.20 < 84\varepsilon$$

'd'can also be found as D-2(t_f-r_o). However there is some difference. Comment !
Hence web is a plastic element.

Hence section is plastic section.

Check for shear capacity of section *(Clause 8.4 of IS: 800-2007)*

Factored design shear force, V = 94.5 *kN*

Design shear strength, $V_d = \frac{f_y h t_w}{\gamma_{m0}\sqrt{3}}$

$$= \frac{250x300x7.5}{1.10x\sqrt{3}} = 295.23\ kN > 94.5kN$$

$0.6V_d$ = (0.6x295.23) = 177.14 *kN* > 94.5 *kN* ***Hence Safe…***

Design bending capacity of the section *(Clause 8.2.1.2 of IS:800-2007)*

$$M_d = \beta_b Z_p f_y / \gamma_{m0} \le 1.2 Z_e f_y / \gamma_{m0}$$

$$\beta_b Z_p f_y / \gamma_{m0} = 1.0x651.74x10^3 x250/1.10 = 148.12\ kN\text{-}m$$

$$1.2 Z_e f_y / \gamma_{m0} = 1.2x573.6x10^3 x250/1.10 = 156.44\ kN\text{-}m > 148.12\ kN\text{-}m$$

Hence design bending moment, M_d = 148.12 *kN-m*

Check for web buckling *(Clause 8.7.3.1 of IS: 800-2007)*

Buckling strength of unstiffened web, $F_{cdw} = (b_1 + n_1) t_w f_{cd}$

Assuming stiff bearing length, b_1=100 *mm*

n_1= 300/2 = 150 *mm*; d = 241.5 *mm*

Effective length, = 0.7d = 0.7x241.5= 169.05 *mm*

$r_{min} = \frac{t_w}{\sqrt{12}} = \frac{7.5}{\sqrt{12}} = 2.16$ *mm*

$\frac{l_{eff}}{r_{min}} = \frac{169.05}{2.16} = 78.26$

Form Table 9(c) of IS: 800-2007 f_{cd}

Design compressive stress, f_{cd} = 138.78 *N/mm²*

Strength of the section against web buckling = (100+150)7.5x138.78

= 260.21 *kN* >94.5 *kN* ***Hence safe…***

Check for web bearing (crippling or Crimpling) *(Clause 8.7.4 of IS: 800-2007)*

Design capacity of web in bearing, $F_w = (b_1 + n_2) t_w f_y / \gamma_{m0}$

Assume stiff bearing length, b_1= 100 *mm*

$n_2 = 2.5(d_2) = 2.5(29.25) = 73.12$

$F_w = ((100 + 73.12) x 7.5 x 250) / 1.10 = 295.1$ *kN* > 94.5 *kN* ***Hence safe…***

Check for deflection:

Actual deflection is given by

$$\delta = \frac{5wl^4}{384EI} = \frac{5x20x6000^4}{384x2x10^5 x8603.6x10^4} = 19.61 \text{ mm}$$

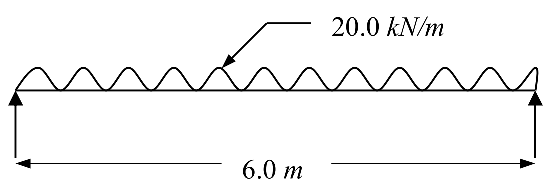
Allowable deflection $\frac{L}{300} = \frac{6000}{300} = 20.0$ *mm* >19.61 *mm* ***Hence safe…***

Working stress method

Effective span, l_{eff} = 6.0 *m*

Assume self-weight of beam=1.0 *kN/m*

Total load = 20+1.0 = 21.0 *kN/m*



$M = \frac{wl^2}{8} = \frac{21.0x6^2}{8} = 94.5$ *kN-m*

$V = \frac{wl}{2} = \frac{21.0x6}{2} = 63.0$ *kN*

Maximum permissible bending stress in tension and compression, $\sigma_{bt} = \sigma_{bc} = 0.66f_y$

$= 0.66x250 = 165$ *N/mm²*

Required section modulus Z_e, $= \frac{M}{\sigma_{bc}} = \frac{94.5x10^6}{165} = 572.72x10^3$ *mm³*

It is found that by section ISMB 300 @ 44.2 *kg/m* will fail due to deflection.

Therefore section ISMB 350 @ 52.4 *kg/m* is provided.

Sectional properties:

Sectional area, A = 6671 *mm²*

Depth of section, D = 350 *mm*

Width of flange, b_f = 140 *mm*

Moment of inertia, I_{zz} =13630.3 *cm⁴*

Moment of inertia, I_{yy} = 537.7 *cm⁴*

Plastic modulus, Z_{pz} =889.57 *cm³*

Depth of wed, d = 288.0 *mm*

Thickness of flange, t_f = 14.2 *mm*

Thickness of web, t_w = 8.1 *mm*

Radius of gyration, r_{zz} = 14.29 *cm*

Radius of gyration, r_{yy} = 2.84 *cm*

Section modulus, Z_{ez} = 778.9 *cm³*

Section modulus, Z_{ey} = 76.8 *cm³*

d_2 = 31.0 *mm*

Design bending strength of ISMB 350 @ 52.4 *kg/m* = $Z_e \times \sigma_{bc} = 778.9x10^3x165$

= 128.52 *kN-m* > 94.5 *kN-m*

Hence safe…

Check for shear

Average shear stress= $\tau_{va\,(cal)} = \frac{V}{t_w h}$

$$= \frac{63.0x10^3}{8.1x350} = 22.22 \text{ N/mm}^2$$

Permissible shear stress, $\tau_{va} = 0.4f_y$

= 0.4x250 = 100 *N/mm²* < 22.22 *N/mm²* ***Hence safe…***

Check for web buckling

Slenderness ratio, λ_w = $\frac{d}{t}\sqrt{3}$

$$= \frac{288.0x\sqrt{3}}{8.1} = 61.58$$

From Table 5.1 of IS: 800-1984, permissible stress σ_{ac} for λ_w = 61.58

σ_{ac} = 120.42 *N/mm²*

Axial stress $\sigma_{ac,\,cal} = \frac{V}{t_w\left(b+n_1\right)} = \frac{63x10^3}{8.1\left(100+175\right)} = 28.28$ *N/mm²* <120.42 *N/mm²*

Hence safe…

Check for web bearing

Maximum permissible bearing stress, $\sigma_p = 0.75f_y$

$$= 0.75x250 = 187.5\ N/mm^2$$

Hence safe…

Web bearing stress = $\dfrac{V}{\left(b + d_2\sqrt{3}\right)t_w}$

$$= \frac{63x10^3}{\left(100 + \left(31.0x\sqrt{3}\right)\right)8.1} = 50.60\ N/mm^2 < 187.5\ N/mm^2$$

Check for deflection

Total load = 20+0.524 = 20.524 kN/*m*

Deflection, $\delta = \dfrac{5wl^4}{384EI}$

$$= \frac{5x20.534x6000^4}{384x2x10^5x13630.3x10^4} = 12.71\ mm$$

Allowable deflection, $\dfrac{Span}{325} = \dfrac{6000}{325} = 18.46\ mm > 12.71\ mm$ ***Hence safe…***

2. **Design a fixed beam to carry a uniformly distributed load of 20 *kN/m*. The effective span of beam is 6.0 *m***

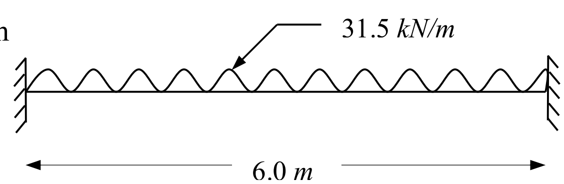
SOLUTION

Limit State Method

Assuming the self weight of beam = 1.0kN/m

Effective span, $l_{eff} = 6.0\ m$

Factored load = 21x1.5 = 31.5 *kN/m*



Max. Bending moment, M = $\dfrac{wl^2}{12} = \dfrac{31.5x6^2}{12} = 94.5\ kN\text{-}m$

Max. Shear force, V= $\dfrac{wl}{2} = \dfrac{31.5x6}{2} = 94.5\ kN$

Plastic Section modulus required

$$Z_p = \frac{M\gamma_{m0}}{f_y} = \frac{94.5x10^6x1.10}{250} = 415.8x10^3\ mm^3$$

Consider section ISLB 275 @ 33.0 *kg/m*

Sectional properties:

Sectional area, A = 4202 mm^2

Depth of section, D = 275 *mm*

Width of flange, b_f = 140 *mm*

Moment of inertia, I_{zz} =5375.3 cm^4

Moment of inertia, I_{yy} = 287 cm^4

Plastic modulus, Z_{pz} =443.09 cm^3

Radius at root, r_1= 14.0 *mm*

d_2 = 25.65 *mm*

Thickness of flange, t_f = 8.8 *mm*

Thickness of web, t_w = 6.4 *mm*

Radius of gyration, r_{zz} = 11.31 *cm*

Radius of gyration, r_{yy} = 2.61 *cm*

Section modulus, Z_{ez} = 392.4 cm^3

Section modulus, Z_{ey} = 41 cm^3

Depth of web, d = 223.7 *mm*

Section classification *Table-2 of code IS: 800-2007*

$$\varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Outstanding flanges:

$$\frac{b}{t_f} = \frac{140}{2x8.8} = 7.95 < 9.4\varepsilon$$

Hence flange is plastic element.

Web of an I-section:

$$\frac{d}{t_w} = \frac{223.7}{8.8} = 25.42 < 84\varepsilon$$

Hence web is a plastic element.

Hence the section is plastic section.

Check for shear capacity of section *(Clause 8.4 of IS: 800-2007)*

Factored design shear force, V = 94.5 *kN*

Design shear strength, $V_d = \frac{f_y h t_w}{\gamma_{m0}\sqrt{3}}$

$$= \frac{250x275x6.4}{1.10x\sqrt{3}} = 230.94\ kN > 94.5\ kN$$

$0.6V_d$ = (0.6x230.94) = 138.56 *kN* > 94.5 *kN* ***Hence Safe…***

Design bending capacity of the section (*Clause 8.2.1.2 of IS:800-2007)*

$M_d = \beta_b Z_p f_y / \gamma_{m0} \leq 1.2 Z_e f_y / \gamma_{m0}$

$\beta_b Z_p f_y / \gamma_{m0} = 1.0 x 443.09 x 10^3 x 250 / 1.10$ = 100.70 *kN-m*

$1.2 Z_e f_y / \gamma_{m0} = 1.2 x 392.4 x 10^3 x 250 / 1.10$ = 107.018 *kN-m* >100.70 *kN-m*

Hence design bending strength, M_d = 100.70 *kN-m*

Total load including its self-weight =20+0.524 = 20.524 *kN/m*

Factored load = (20.524x1.5) = 30.78 *kN/m*

Maximum bending moment, $\dfrac{30.78 x 6^2}{12}$ = 92.34 < 100.70 *kN-m*

Check for web buckling *(Clause 8.7.3.1 of IS: 800-2007)*

Buckling strength of unstiffened web, $F_{cdw} = (b_1 + n_1) t_w f_{cd}$

Stiff bearing length, b_1=100 *mm*

n_1= 275/2 = 137.5 *mm*; d = 223.7 *mm*

Effective length, = 0.7d = 0.7x223.7 = 156.59 *mm*

$r_{min} = \dfrac{t_w}{\sqrt{12}} = \dfrac{6.4}{\sqrt{12}}$ =1.84 *mm*

$\dfrac{l_{eff}}{r_{min}} = \dfrac{156.59}{1.84} = 84.76$

Form Table 9(c) of IS: 800-2007 f_{cd}

Design compressive stress, f_{cd} = 128.86 N/mm^2

Strength of the section against web buckling = ((100+137.5)6.4x128.86

= 195.86 *kN* > 94.5 *kN* ***Hence safe…***

Check for web bearing *(Clause 8.7.4 of IS: 800-2007)*

Design capacity of web in bearing, $F_w = (b_1 + n_2) t_w f_y / \gamma_{m0}$

Stiff bearing length, b_1= 100 *mm*

$n_2 = 2.5(t_f + r_1) = 2.5(8.8 + 14.0) = 57.0$

$F_w = ((100 + 57.0) x 6.4 x 250) / 1.10$ = 228.36 *kN* > 94.5 *kN* ***Hence safe…***

Check for deflection

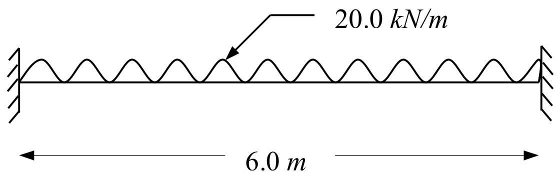
$$\delta = \frac{5wl^4}{384EI} = \left(\frac{5x20x6000^4}{384x2x10^5x5375.3x10^4}\right)0.50 = 15.69\ mm$$

Allowable deflection $\frac{L}{300} = \frac{6000}{300} = 20.0\ mm$ >15.69 *mm* ***Hence safe…***

Working stress method

Assume self-weight of beam = 1.0kN/m

Effective span, l_{eff} = 6.0 *m*



$M = \frac{wl^2}{12} = \frac{21.0x6^2}{12} = 63\ kN\text{-}m$

$V = \frac{wl}{2} = \frac{21.0x6}{2} = 63\ kN\text{-}m$

Maximum permissible bending stress in tension and compression, $\sigma_{bt} = \sigma_{bc} = 0.66f_y$

= 0.66x250 = 165 *N/mm²*

Required section modulus Z, $= \frac{M}{\sigma_{bc}} = \frac{63x10^6}{165} = 381.82x10^3\ mm^3$

Consider section ISLB 275 @ 33.0 *kg/m*

Sectional properties:

Sectional area, A = 4202 mm^2

Depth of section, D = 275 *mm*

Width of flange, b_f = 140 *mm*

Moment of inertia, I_{zz} =5375.3 cm^4

Moment of inertia, I_{yy} = 287 cm^4

Plastic modulus, Z_{pz} =443.09 cm^3

Thickness of flange, t_f = 8.8 *mm*

Thickness of web, t_w = 6.4 *mm*

Radius of gyration, r_{zz} = 11.31 *cm*

Radius of gyration, r_{yy} = 2.61 *cm*

Section modulus, Z_{ez} = 392.4 cm^3

Section modulus, Z_{ey} = 41 cm^3

Depth of web, d = 223.7 *mm*

d_2 = 25.65 *mm*

Total load including self-weight = 20 + 0.33 = 20.33 *kN/m*

Maximum bending moment, $\frac{20.33x6^2}{8} = 91.48\ kN\text{-}m$

Design bending strength of ISLB 275 @ 33.0 *kg/m* = $Z_e x\ \sigma_{bc}$ = $392.4x10^3x165$

= 64.74 *kN-m* >63 *kN-m*

Hence safe…

Check for shear

Average shear stress= $\tau_{va\,(cal)} = \frac{V}{t_w h}$

$$= \frac{63 x 10^3}{6.4 x 275} = 35.79 \; N/mm^2$$

Permissible shear stress, $\tau_{va} = 0.4 f_y$

$= 0.4 x 250 = 100 \; N/mm^2 < 35.79 \; N/mm^2$ ***Hence safe…***

Check for web buckling

Slenderness ratio, $\lambda_w = \frac{d}{t}\sqrt{3}$

$$= \frac{223.7 x \sqrt{3}}{6.4} = 60.54$$

From Table 5.1 of IS: 800-1984, permissible stress σ_{ac} *for* $\lambda_w = 60.54$

$\sigma_{ac} = 121.46 \; N/mm^2$

Axial stress $\sigma_{ac,\,cal} = \frac{V}{t_w (b + n_1)} = \frac{63 x 10^3}{6.4(100 + 137.5)} = 41.44 \; N/mm^2 < 120.42 \; N/mm^2$

Hence safe…

Check for web bearing

Maximum permissible bearing stress, $\sigma_p = 0.75 f_y$

$= 0.75 x 250 = 187.5 \; N/mm^2$

Web bearing stress = $\frac{V}{(b + d_2\sqrt{3}) t_w}$

$$= \frac{63 x 10^3}{(100 + (25.85 x \sqrt{3}))6.4} = 67.99 \; N/mm^2 < 187.5 \; N/mm^2$$

Hence safe…

Check for deflection

Total load = 20+0.33 = 20.33 *kN/m*

Deflection, $\delta = \left(\frac{5wl^4}{384EI}\right)0.50$

$$= \left(\frac{5 x 20.33 x 6000^4}{384 x 2 x 10^5 x 5375.3 x 10^4}\right)0.5 = 15.95 \; mm$$

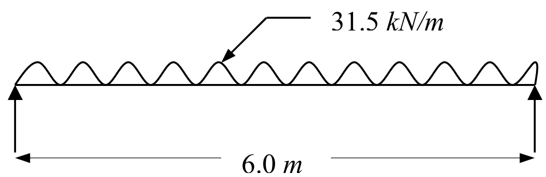
Allowable deflection, $\frac{Span}{325} = \frac{6000}{325} = 18.46 \; mm > 15.95 \; mm$ ***Hence safe…***

LATERALLY UNSUPPORTED BEAMS

3. Design a simply supported beam to carry a uniformly distributed load of 20 *kN/m*. The effective span of beam is 6.0 *m*

SOLUTION

Limit State Method



Assuming the self weight of beam = 1.0 *kN/m*

Total load = 20+1.0 = 21.0 *kN/m*

Factored load = 21x1.5 = 31.5 *kN/m*

Bending moment, $M = \frac{wl^2}{8} = \frac{31.5x6^2}{8} = 141.75$ *kN-m*

Shear force, $V = \frac{wl}{2} = \frac{31.5x6}{2} = 94.5$ *kN*

Plastic Section modulus required

$Z_p = \frac{M\gamma_{m0}}{f_y} = \frac{141.75x10^6 x1.10}{250} = 623.70x10^3$ mm^3

Consider section ISMB 350 @ 52.4 *kg/m*

Sectional properties:

Sectional area, A = 6671 mm^2

Depth of section, D = 350 *mm*

Width of flange, b_f = 140 *mm*

Moment of inertia, I_{zz} =13630.3 cm^4

Moment of inertia, I_{yy} = 537.7 cm^4

Plastic modulus, Z_{pz} =889.57 cm^3

Radius at root, r_1= 14.0 *mm*

d_2 = 31.0 *mm*

Thickness of flange, t_f = 14.2 *mm*

Thickness of web, t_w = 8.1 *mm*

Radius of gyration, r_{zz} = 14.29 *cm*

Radius of gyration, r_{yy} = 2.84 *cm*

Section modulus, Z_{ez} = 778.9 cm^3

Section modulus, Z_{ey} = 76.8 cm^3

Depth of web, d = 288.0 *mm*

Section classification *Table-2 of code IS: 800-2007*

$$\varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Outstanding flanges:

$$\frac{b}{t_f} = \frac{140}{2x14.2} = 4.93 < 9.4\varepsilon$$

Hence flange is plastic element.

Web of an I-section:

$$\frac{d}{t_w} = \frac{288.0}{8.1} = 36.24 \ < \ 84\varepsilon$$

Hence web is a plastic element.

Hence section is plastic section.

To determine the bending strength

Elastic lateral buckling, $M_{cr} = \sqrt{\left[\left(\frac{\pi^2 EI_y}{(L_{LT})^2}\right)\left(GI_t + \frac{\pi^2 EI_w}{(L_{LT})^2}\right)\right]}$ (3.1)

Torsional constant, $I_t = \sum b_i t_i^3/3$

$$= \left[\left(\frac{2x140x14.2^3}{3}\right) + \left(\frac{335.8x8.1^3}{3}\right)\right] = 326.726x10^3 \, mm^4 \quad \text{(a)}$$

Warping constant, $I_w = (1-\beta_f)\beta_f I_y h_f^2$

$\beta_f = 0.50$

$h_f = (D-t_f) = (350-14.2) = 335.8$ *mm*

$$\therefore I_w = (1-0.5)x0.5x537.7x10^4 x(335.8)^2 = 1.515x10^{11} \, mm^4 \quad \text{(b)}$$

$$G = \frac{E}{2(1+\mu)} = \frac{2x10^5}{2(1+0.3)} = 76.923x10^3 \ N/mm^2 \quad \text{(c)}$$

Substituting (a), (b), (c) in equation 3.1

$$M_{cr} = \sqrt{\left[\left(\frac{\pi^2 x2x10^5 x537.7x10^4}{(0.7x6000)^2}\right)\left((76.923x10^3 x326.726x10^3) + \left(\frac{\pi^2 x2x10^5 x1.515x10^{11}}{(0.7x6000)^2}\right)\right)\right]}$$

= 159.14 *kN-m*

Non-dimensional slenderness ratio, $\lambda_{LT1} = \sqrt{\frac{\beta_b Z_{pz} f_y}{M_{cr}}} \le \lambda_{LT2} = \sqrt{\frac{1.2 Z_{ez} f_y}{M_{cr}}}$

$$\lambda_{LT1} = \sqrt{\frac{1.0x889.57x10^3 x250}{159.147x10^6}} = 1.18$$

$$\lambda_{LT2} = \sqrt{\frac{1.2x778.9x10^3x250}{159.147x10^6}} = 1.21$$

$\therefore\ \lambda_{LT} = 1.21$

$$\phi_{LT} = 0.5[1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^2]$$

$$= 0.5[1+0.21(1.21-0.2)+1.21^2] = 1.30$$

Bending stress reduction factor, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT}+[\phi_{LT}^2-\lambda_{LT}^2]^{0.5}\right\}}$

$$= \frac{1}{\left\{1.30+[1.30^2-1.21^2]^{0.5}\right\}} = 0.541 \le 1.0$$

Design bending compressive stress, $f_{bd} = \dfrac{\chi_{LT} f_y}{\gamma_{mo}} = \dfrac{0.541x250}{1.10} = 123.04\ kN/mm^2$

Design bending strength, $M_d = \beta_b Z_p f_{bd}$

$= 1.0x889.57x10^3x123.042 = 109.48\ kN-m$ ***Hence unsafe…***

Higher section has to be provided.

Consider section ISMB 400 at 61.6 *kg/m*

Sectional properties:

Sectional area, A = 7846 mm^2

Depth of section, D = 400 *mm*

Width of flange, b_f = 140 *mm*

Moment of inertia, I_{zz} =20458.4 cm^4

Moment of inertia, I_{yy} = 622.1 cm^4

Plastic modulus, Z_{pz} = 1176.18 cm^3

Depth of web, d = 334.4 *mm*

Thickness of flange, t_f = 16.0 *mm*

Thickness of web, t_w = 8.9 *mm*

Radius of gyration, r_{zz} = 16.15 *cm*

Radius of gyration, r_{yy} = 2.82 *cm*

Section modulus, Z_{ez} = 1022.9 cm^3

Section modulus, Z_{ey} = 88.9 cm^3

d_2 = 32.80 *mm*

To determine design bending strength

Elastic lateral buckling, $M_{cr} = \sqrt{\left[\left(\dfrac{\pi^2 EI_y}{(L_{LT})^2}\right)\left(GI_t+\dfrac{\pi^2 EI_w}{(L_{LT})^2}\right)\right]}$

Torsional constant, $I_t = \sum b_i t_i^3/3$

$$= \left[\left(\frac{2x140x16.0^3}{3}\right)+\left(\frac{391.1x8.9^3}{3}\right)\right] = 474.19x10^3\ mm^4$$

Warping constant, $I_w = (1-\beta_f)\beta_f I_y h_f^2$

$\beta_f = 0.50$

$h_f = (D-t_f) = (400-16.0) = 384.0$ *mm*

$\therefore I_w = (1-0.5) x 0.5 x 622.1 x 10^4 x (384.0)^2 = 2.293 x 10^{11}$ *mm⁴*

$$G = \frac{E}{2(1+\mu)} = \frac{2x10^5}{2(1+0.3)} = 76.923x10^3 \ N/mm^2$$

Substituting (a), (b), (c) in equation 3.2

$$M_{cr} = \sqrt{\left[\left(\frac{\pi^2 x 2x10^5 x 622.1x10^4}{(6000)^2}\right)\left(\left(76.923x10^3 x 472.53x10^3\right) + \left(\frac{\pi^2 x 2x10^5 x 2.293x10^{11}}{(6000)^2}\right)\right)\right]}$$

= 207.768 *kN-m*

Non-dimensional slenderness ratio, $\lambda_{LT1} = \sqrt{\frac{\beta_b Z_{pz} f_y}{M_{cr}}} \leq \lambda_{LT2} = \sqrt{\frac{1.2 Z_{ez} f_y}{M_{cr}}}$

$$\lambda_{LT1} = \sqrt{\frac{1.0x1176.18x10^3 x250}{207.768x10^6}} = 1.186$$

$$\lambda_{LT2} = \sqrt{\frac{1.2x1020.0x10^3 x250}{207.768x10^6}} = 1.213$$

$\therefore \lambda_{LT} = 1.186$

$$\phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

$$= 0.5[1 + 0.21(1.186 - 0.2) + 1.186^2] = 1.311$$

Bending stress reduction factor, $\chi_{LT} = \frac{1}{\left\{\phi_{LT} + [\phi_{LT}^2 - \lambda_{LT}^2]^{0.5}\right\}}$

$$= \frac{1}{\left\{1.311 + [1.311^2 - 1.186^2]^{0.5}\right\}} = 0.536 \leq 1.0$$

Design bending compressive stress, $f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{mo}}$

$$= \frac{0.536x250}{1.10} = 121.946 \ N/mm^2$$

Design bending strength, $M_d = \beta_b Z_p f_{bd}$

$= 1.0x1176.18x10^3 x121.946 = 143.43$ *kN – m* <141.75 *kN-m*

Check for web buckling *(Clause 8.7.3.1 of IS: 800-2007)*

Buckling strength of unstiffened webs, $F_{cdw} = (b_1 + n_1)\, t_w f_{cd}$

Stiff bearing length, b_1=100 *mm*

n_1= 400/2 = 200 *mm*; d = 334.4 *mm*

Effective length, = 0.7d = 0.7x334.4 = 234.08 *mm*

$r_{min} = \frac{t_w}{\sqrt{12}} = \frac{8.9}{\sqrt{12}} = 2.57$ *mm*

$\frac{l_{eff}}{r_{min}} = \frac{234.08}{2.57} = 91.11$

Form Table 9(c) of IS: 800-2007 f_{cd}

Design compressive stress, f_{cd} = 119.45 *N/mm²*

Strength of the section against web buckling = ((100+200)8.1x119.45

= 290.25 *kN* >94.5 *kN* ***Hence safe…***

Check for web bearing *(Clause 8.7.4 of IS: 800-2007)*

Design capacity of web in bearing, $F_w = (b_1 + n_2)\, t_w f_y / \gamma_{m0}$

Stiff bearing length, b_1= 100 *mm*

$n_2 = 2.5(d_2) = 2.5(32.80) = 82.0$

$F_w = ((100 + 82.0)x8.9x250)/1.10 = 368.14$ *kN* > 94.5 *kN* ***Hence safe…***

Check for deflection

$$\delta = \left(\frac{5wl^4}{384EI}\right)0.5 = \left(\frac{5x20x6000^4}{384x2x10^5x20458.4x10^4}\right)0.50 = 4.12 \text{ mm}$$

Allowable deflection $\frac{L}{300} = \frac{6000}{300} = 20.0$ *mm* >4.12 *mm* ***Hence safe…***

Working stress method

From Clause 6.2.2 of IS: 800-1984

Effective span, l_{eff} = 6.0 *m*

$M = \frac{wl^2}{8} = \frac{21.0x6^2}{8}$ **= 64.5 *kN-m***

$V = \frac{wl}{2} = \frac{21.0x6}{2} = 63.0\ kN$

Maximum permissible bending stress in tension and compression, $\sigma_{bt} = \sigma_{bc} = 0.66f_y$

$= 0.66x250 = 165\ N/mm^2$

Assume allowable bending stress, $\sigma_{bc} = 95\ N/mm^2$

Required section modulus Z, $= \frac{M}{\sigma_{bc}} = \frac{94.5x10^6}{95} = 994.74x10^3\ mm^3$

Consider ISMB 400 at 61.6 *kg/m*

Sectional properties:

Sectional area, A = 7846 mm^2

Depth of section, D = 400 *mm*

Width of flange, b_f = 140 *mm*

Moment of inertia, I_{zz} =20458.4 cm^4

Moment of inertia, I_{yy} = 622.1 cm^4

Plastic modulus, Z_{pz} = 1176.18 cm^3

Depth of web, d = 334.4 *mm*

Thickness of flange, t_f = 16.0 *mm*

Thickness of web, t_w = 8.9 *mm*

Radius of gyration, r_{zz} = 16.15 *cm*

Radius of gyration, r_{yy} = 2.82 *cm*

Section modulus, Z_{ez} = 1022.9 cm^3

Section modulus, Z_{ey} = 88.9 cm^3

d_2 = 32.80 *mm*

Permissible bending compressive stress, σ_{bc}

$d_1 = 400 - 2(16.0) = 368.0\ mm$

Mean thickness of compression flange, T = 16.0 *mm*

Web thickness, t = 8.9 *mm*

Overall depth of web, D = 400 *mm*

$D/T = 400/16.0 = 25.00$

$T/t = 16.0/8.9 = 1.79 \leq 2.0$

$d_1/t = 368.0/8.9 = 41.35 \leq 85$

Effective length of compression flange, l_{eff} = 0.7x6000 = 4200 *mm*

$l/r_y = 4200/28.2 = 148.936$

From Table 6.1B of IS: 800-1984, σ_{bc} is calculated by double interpolation.

kL/r	D/T
	25

140	103
148.936	98.532
150	98

∴ Bending compressive stress, σ_{bc} = 98.532 *N/mm²*

Moment of resistance, M_r = 98.532x1022.9x10³

= 100.788 *kN-m* >>64.5 *kN-m* ***Hence safe…***

Therefore a lower section can be adopted.

Check for shear

Average shear stress= $\tau_{va\,(cal)} = \frac{V}{t_w h}$

$$= \frac{64.5x10^3}{8.9x400} = 18.12\ N/mm^2$$

Permissible shear stress, $\tau_{va} = 0.4f_y$

= 0.4x250 = 100 *N/mm² < 26.03 N/mm²* ***Hence safe…***

Check for web buckling

Slenderness ratio, $\lambda_w = \frac{d}{t}\sqrt{3}$

$$= \frac{334.4x\sqrt{3}}{8.9} = 65.07$$

From Table 5.1 of IS: 800-1984, permissible stress σ_{ac} for λ_w = 65.07
σ_{ac} = 116.93 *N/mm²*

Axial stress $\sigma_{ac,\,cal} = \frac{V}{t_w(b+n_1)} = \frac{63x10^3}{8.9(100+200)} = 23.59\ N/mm^2 < 116.93\ N/mm^2$

Hence safe…

Check for web bearing

Maximum permissible bearing stress, $\sigma_p = 0.75f_y$

= 0.75x250 = 187.5 *N/mm²*

Web bearing stress = $\frac{V}{(b+d_2\sqrt{3})t_w}$

$$= \frac{63x10^3}{(100+(32.8x\sqrt{3}))8.9} = 45.14\ N/mm^2 < 187.5\ N/mm^2$$

Check for deflection

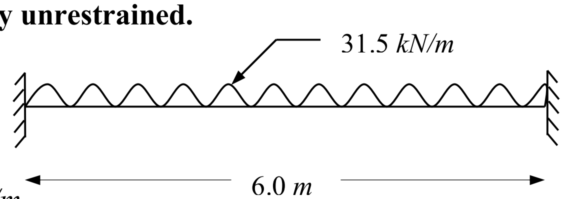
Total load = 20+0.61 = 20.61 *kN/m*

Deflection, $\delta = \left(\frac{5wl^4}{384EI}\right)$

$$= \left(\frac{5x20.61x6000^4}{384x2x10^5x20458.4x10^4}\right) = 8.50\ mm$$

Allowable deflection, $\frac{Span}{325} = \frac{6000}{325} = 18.46\ mm > 8.5\ mm$ ***Hence safe…***

4. **Design a fixed beam to carry a uniformly distributed load of 20 *kN/m*. the effective span of beam is 6.0m. Beam is laterally unrestrained.**



SOLUTION

Limit State Method

Assuming the self weight of beam = 1.0 *kN/m*

Effective span, l_{eff} = 6.0 *m*

Factored load = 21x1.5 = 31.5 *kN/m*

Max. Bending moment, $M = \frac{wl^2}{12} = \frac{31.5x6^2}{12} = 94.5\ kN\text{-}m$

Max. Shear force, $V = \frac{wl}{2} = \frac{31.5x6}{2} = 94.5\ kN$

Plastic Section modulus required

$$Z_p = \frac{M\gamma_{m0}}{f_y} = \frac{94.5x10^6x1.10}{250} = 415.8x10^3\ mm^3$$

Consider section ISLB 350 @ 49.5 *kg/m*

Sectional properties:

Sectional area, A = 6301 mm^2

Depth of section, D = 350 *mm*

Width of flange, b_f = 165 *mm*

Moment of inertia, I_{zz} =13158.3 cm^4

Moment of inertia, I_{yy} = 631.9cm^4

Plastic modulus, Z_{pz} = 851.11 cm^3

Radius at root = 16.0 *mm*

Thickness of flange, t_f = 11.4 *mm*

Thickness of web, t_w = 7.4 *mm*

Radius of gyration, r_{zz} = 14.45 *cm*

Radius of gyration, r_{yy} = 3.17 *cm*

Section modulus, Z_{ez} = 751.9 cm^3

Section modulus, Z_{ey} = 76.6 cm^3

Depth of section, d = 288.3 *mm*

$d_2 = 30.85\ mm$

Section classification *Table-2 of code IS: 800-2007*

$$\varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Outstanding flanges:

$$\frac{b}{t_f} = \frac{165}{2x11.4} = 7.23 < 9.4\varepsilon$$

Hence flange is plastic element.

Web of an I-section:

$$\frac{d}{t_w} = \frac{288.3}{7.4} = 38.96 < 84\varepsilon$$

Hence web is a plastic element.

Hence the section is plastic section.

Check for shear capacity of section *(Clause 8.4 of IS: 800-2007)*

Factored design shear force, V = 94.5 *kN*

Design shear strength, $V_d = \frac{f_y h t_w}{\gamma_{m0}\sqrt{3}}$

$$= \frac{250x350x7.4}{1.10x\sqrt{3}} = 339.85\ kN > 94.5\ kN$$

To determine design bending strength

Elastic lateral buckling, $M_{cr} = \sqrt{\left[\left(\frac{\pi^2 EI_y}{(L_{LT})^2}\right)\left(GI_t + \frac{\pi^2 EI_w}{(L_{LT})^2}\right)\right]}$

Torsional constant, $I_t = \sum b_i t_i^3/3$

$$= \left[\left(\frac{2x165x11.4^3}{3}\right) + \left(\frac{338.6x7.4^3}{3}\right)\right] = 208.71x10^3\ mm^4$$

Warping constant, $I_w = (1-\beta_f)\beta_f I_y h_f^2$

$\beta_f = 0.50$

$h_f = (D-t_f) = (350-11.4) = 338.6$ *mm*

$\therefore I_w = (1-0.5)x0.5x622.1x10^4x(338.6)^2 = 1.811x10^{11}$ *mm⁴*

$G = \frac{E}{2(1+\mu)} = \frac{2x10^5}{2(1+0.3)} = 76.923x10^3$ *N/mm²*

Substituting in equation

$$M_{cr} = \sqrt{\left[\left(\frac{\pi^2 x2x10^5 x631.9x10^4}{(0.7x6000)^2}\right)\left((76.923x10^3 x208.706x10^3) + \left(\frac{\pi^2 x2x10^5 x1.811x10^{11}}{(0.7x6000)^2}\right)\right)\right]}$$

$= 160.26$ *kN-m*

Non-dimensional slenderness ratio, $\lambda_{LT1} = \sqrt{\frac{\beta_b Z_{pz} f_y}{M_{cr}}} \leq \lambda_{LT2} = \sqrt{\frac{1.2 Z_{ez} f_y}{M_{cr}}}$

$$\lambda_{LT1} = \sqrt{\frac{1.0x851.110x10^3 x250}{160.26x10^6}} = 1.15$$

$$\lambda_{LT2} = \sqrt{\frac{1.2x751.9x10^3 x250}{160.26x10^6}} = 1.18$$

$\therefore \lambda_{LT} = 1.18$

$$\phi_{LT} = 0.5[1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^2]$$

$$= 0.5[1+0.21(1.18-0.2)+1.18^2] = 1.26$$

Bending stress reduction factor, $\chi_{LT} = \frac{1}{\{\phi_{LT} + [\phi_{LT}^2 - \lambda_{LT}^2]^{0.5}\}}$

$$= \frac{1}{\{1.26+[1.26^2-1.18^2]^{0.5}\}} = 0.56 \leq 1.0$$

Design bending compressive stress, $f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{mo}}$

$$= \frac{0.56x250}{1.10} = 127.46 \text{ N/mm}^2$$

Design bending strength, $M_d = \beta_b Z_{pz} f_{bd}$

$$= 1.0x851.11x10^3 x127.46 = 108.48 \ kN-m > 94.5 \ kN\text{-}m$$

Check for web buckling *(Clause 8.7.3.1 of IS: 800-2007)*

Buckling strength of unstiffened web, $F_{cdw} = (b_1 + n_1) t_w f_{cd}$

Stiff bearing length, b_1=100 *mm*

n_1= 350/2 = 175 *mm*; d = 288.3 *mm*

Effective length, = 0.7d = 0.7x288.3 = 201.81 *mm*

$r_{min} = \frac{t_w}{\sqrt{12}} = \frac{7.4}{\sqrt{12}} = 2.14$

$\frac{l_{eff}}{r_{min}} = \frac{201.81}{2.14} = 94.30$

Form Table 9(c) of IS: 800-2007, f_{cd}

Design compressive stress, f_{cd} = 114.97 *N/mm²*

Strength of the section against web buckling = (100+175)7.4x114.97

= 233.96 *kN* >94.5 *kN* ***Hence safe…***

Check for web bearing *(Clause 8.7.4 of IS: 800-2007)*

Design capacity of web in bearing, $F_w = (b_1 + n_2) t_w f_y / \gamma_{m0}$

Assume stiff bearing length, b_1= 100 *mm*

$n_2 = 2.5(d_2) = 2.5 \text{ x}30.85 = 77.12$

$F_w = ((100 + 77.12)\text{x}7.4\text{x}250)/1.10 = 297.89\ kN > 94.5\ kN$ ***Hence safe…***

Check for deflection

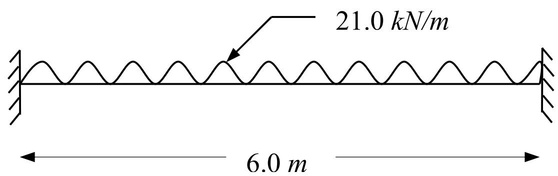
$$\delta = \left(\frac{5wl^4}{384EI}\right)0.5 = \left(\frac{5\text{x}20\text{x}6000^4}{384\text{x}2\text{x}10^5\text{x}13158.3\text{x}10^4}\right)0.50 = 6.41\ mm$$

Allowable deflection $\frac{L}{300} = \frac{6000}{300} = 20.0\ mm > 6.41\ mm$ ***Hence safe…***

Working stress method

From Clause 6.2.2 of IS: 800-1984

Effective span, l_{eff} = 6.0 *m*



$M = \frac{wl^2}{12} = \frac{21.0\text{x}6^2}{12} = 63$ k*N-m*

$V = \frac{wl}{2} = \frac{21.0\text{x}6}{2} = 63$ *kN-m*

Maximum permissible bending stress in tension and compression, $\sigma_{bt} = \sigma_{bc} = 0.66 f_y$

$= 0.66x250 = 165\ N/mm^2$

Required section modulus Z, $= \frac{M}{\sigma_{bc}} = \frac{63x10^6}{165} = 381.82x10^3\ mm^3$

Consider section ISLB 350 at 49.5 *kN/m*

Sectional properties

Sectional area, A = 6301 mm^2

Thickness of flange, t_f = 11.4 *mm*

Depth of section, D = 350 *mm*

Thickness of web, t_w = 7.4 *mm*

Width of flange, b_f = 165 *mm*

Radius of gyration, r_{zz} = 14.45 *cm*

Moment of inertia, I_{zz} =13158.3 cm^4

Radius of gyration, r_{yy} = 3.17 *cm*

Moment of inertia, I_{yy} = 631.9cm^4

Section modulus, Z_{ez} = 751.9 cm^3

Plastic modulus, Z_{pz} = 851.11 cm^3

Section modulus, Z_{ey} = 76.6 cm^3

Radius at root = 16.0 *mm*

Depth of section, d = 288.3 *mm*

d_2 = 30.85 *mm*

Permissible bending compressive stress, σ_{bc}

$d_1 = 350 - 2(11.4) = 327.2\ mm$

Mean thickness of compression flange, T = 11.4 *mm*

Web thickness, t = 7.4 *mm*

Overall depth of web, D = 350 *mm*

$D/T = 350/11.4 = 30.70$

$T/t = 11.4/7.4 = 1.54 \le 2.0$

$d_1/t = 327.2/7.4 = 44.21 \le 85$

Effective length of compression flange, l_{eff} = 0.7x6000 = 4200 *mm*

$l/r_y = 4200/31.7 = 132.49$

From Table 6.1B of IS: 800-1984, σ_{bc} *is calculated by double interpolation*

kL/r	D/T		
	30	30.70	35.0
130	103	102.44	99
132.49		100.98	
140	97	96.58	94

$\therefore$ Bending compressive stress, $\sigma_{bc} = 100.98$ *N/mm²*

Moment of resistance, $M_r = 100.98 x 751.9 x 10^3$

$= 75.93$ *kN-m* > 63 k*N-m* ***Hence safe…***

Check for shear

Average shear stress= $\tau_{va\,(cal)} = \frac{V}{t_w h} = \frac{63.0x10^3}{7.4x350} = 24.32$ *N/mm²*

Permissible shear stress, $\tau_{va} = 0.4 f_y$

$= 0.4x250 = 100$ *N/mm² >24.32 N/mm²* ***Hence safe…***

Check for web buckling

Slenderness ratio, $\lambda_w = \frac{d}{t}\sqrt{3}$

$= \frac{288.3x\sqrt{3}}{7.4} = 67.48$

From Table 5.1 of IS: 800-1984, permissible stress σ_{ac} for $\lambda_w = 67.48$

$\sigma_{ac} = 114.52$ *N/mm²* ; $n_1 = 350/2 = 175$ *mm*

Axial stress $\sigma_{ac,\,cal} = \frac{V}{t_w\left(b+n_1\right)} = \frac{63x10^3}{7.4\left(100+175\right)} = 30.96$ *N/mm²* <114.52 *N/mm²*

Hence safe…

Check for web bearing

Maximum permissible bearing stress, $\sigma_p = 0.75 f_y$

$= 0.75x250 = 187.5$ *N/mm²*

Web bearing stress = $\frac{V}{\left(b+d_2\sqrt{3}\right)t_w}$

$= \frac{63x10^3}{\left(100+\left(30.85x\sqrt{3}\right)\right)7.4} = 55.48$ *N/mm²* < 187.5 N/mm²

Check for deflection

Total load = 20+0.48= 20.48 *kN/m*

Deflection, $\delta = \left(\frac{5wl^4}{384EI}\right)0.50$

$= \left(\frac{5x20.48x6000^4}{384x2x10^5x13158.3x10^4}\right)0.5 = 6.56$ *mm*

Allowable deflection, $\frac{Span}{325} = \frac{6000}{325} = 18.46\ mm > 6.56\ mm$ ***Hence safe…***

5. Calculate the moment carrying capacity of a laterally unrestrained beam of length 3.0 *m* if the following sections are used ISLB300 & ISHB300

SOLUTION

I. ISLB 300 @ 37.7 *kg/m*

Sectional properties

Sectional area, A = 4808 mm^2

Depth of section, D = 300 *mm*

Width of flange, b_f = 150 *mm*

Moment of inertia, I_{zz} = 7332.9 cm^4

Moment of inertia, I_{yy} =376.2 cm^4

Plastic modulus, Z_{pz} = 554.32 cm^3

Radius at root, r_1= 15.0 *m*

Thickness of flange, t_f = 9.4 *mm*

Thickness of web, t_w = 6.7 *mm*

Radius of gyration, r_{zz} = 12.35 *cm*

Radius of gyration, r_{yy} = 2.8 *cm*

Section modulus, Z_{ez} = 488.9 cm^3

Section modulus, Z_{ey} = 50.2 cm^3

Depth of web, d = 245.1 *mm*

Section classification *Table-2 of code IS: 800-2007*

$$\varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Outstanding flanges:

$$\frac{b}{t_f} = \frac{150}{2x9.4} = 7.98 < 9.4\varepsilon$$

Hence flange is plastic element.

Web of an I-section:

$$\frac{d}{t_w} = \frac{245.1}{6.7} = 36.58 < 84\varepsilon$$

Hence web is a plastic element.

Hence section is plastic section.

Method-1

To determine moment carrying capacity of section

Elastic lateral buckling, $M_{cr} = \sqrt{\left[\left(\frac{\pi^2 EI_y}{(L_{LT})^2}\right)\left(GI_t + \frac{\pi^2 EI_w}{(L_{LT})^2}\right)\right]}$

Torsional constant, $I_t = \sum b_i t_i^3 / 3$

$$= \left[\left(\frac{2x150x9.4^3}{3}\right) + \left(\frac{290.6x6.7^3}{3}\right)\right] = 112.192x10^3 \, mm^4$$

Warping constant, $I_w = (1-\beta_f)\beta_f I_y h_f^2$

$$I_{fc} = I_{ft} = \frac{9.4x150^3}{12} = 2.643x10^6 \ mm^4$$

$$\beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} = \frac{2.623x10^6}{(2.623x10^6 + 2.623x10^6)} = 0.50$$

$h_f = (D-t_f) = (300-9.4) = 290.6$ *mm*

$$\therefore I_w = (1-0.5)x0.5x376.2x10^4x(290.6)^2 = 7.942x10^{10} \, mm^4$$

$$G = \frac{E}{2(1+\mu)} = \frac{2x10^5}{2(1+0.3)} = 76.923x10^3 \ N/mm^2$$

Substituting (a), (b), (c), (d) in Equation (2.1)

$$M_{cr} = \sqrt{\left[\left(\frac{\pi^2 x2x10^5 x376.2x10^4}{(3000)^2}\right)\left((76.923x10^3 x112.19x10^3) + \left(\frac{\pi^2 x2x10^5 x7.942x10^{10}}{(3000)^2}\right)\right)\right]}$$

= 146.60 *kN-m*

Non-dimensional slenderness ratio, $\lambda_{LT1} = \sqrt{\frac{\beta_b Z_{pz} f_y}{M_{cr}}} \le \lambda_{LT2} = \sqrt{\frac{1.2 Z_{ez} f_y}{M_{cr}}}$

$$\lambda_{LT1} = \sqrt{\frac{1.0x554.32x10^3 x250}{146.604x10^6}} = 0.972$$

$$\lambda_{LT2} = \sqrt{\frac{1.2x488.9x10^3 x250}{146.604x10^6}} = 1.00$$

$\therefore \lambda_{LT} = 0.972$

$$\phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

$$= 0.5[1 + 0.21(0.972 - 0.2) + 0.972^2]$$

$$= 1.053$$

Bending stress reduction factor, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT} + [\phi_{LT}^2 - \lambda_{LT}^2]^{0.5}\right\}}$

$$= \frac{1}{\left\{1.053 + [1.053^2 - 0.972^2]^{0.5}\right\}} = 0.685 \le 1.0$$

Design bending compressive stress, $f_{bd} = \dfrac{\chi_{LT} f_y}{\gamma_{mo}}$

$$= \frac{0.685x250}{1.10} = 155.681$$

Design bending strength, $M_d = \beta_b Z_p f_{bd}$

$$= 1.0x554.32x10^3 x155.681 = 86.297\ kN-m$$

Method-2

Using the simplified equation which can be used for standard rolled I- section

Elastic lateral buckling moment, $M_{cr} = \dfrac{\pi^2 E I_y h_f}{2L_{LT}^2}\left[1 + \dfrac{1}{20}\left(\dfrac{L_{LT}/r_{yy}}{h_f/t_f}\right)^2\right]^{0.5}$

Assuming the member is torsional fully restraint & warping not restrained in both flanges *(from Table-2 of code IS: 800-2007)*

Effective length for lateral torsional buckling, L_{LT} =KxL = 1.0x3.0 =3.0 *m*

Center-to-center distance between flange, $h_f = (D - t_f)$

= (300 – 9.4) = 290.6 *mm*

$$M_{cr} = \frac{\pi^2 x2x10^5 x376.2x10^4 x290.6}{2x3000^2}\left[1 + \frac{1}{20}\left(\frac{3000/28}{290.6/9.4}\right)^2\right]^{0.5} = 151.66\ kN\text{-}m$$

Non-dimensional slenderness ratio, $\lambda_{LT1} = \sqrt{\dfrac{\beta_b Z_{pz} f_y}{M_{cr}}} \le \lambda_{LT2} = \sqrt{\dfrac{1.2 Z_{ez} f_y}{M_{cr}}}$

$$\lambda_{LT1} = \sqrt{\frac{1.0x554.32x10^3 x250}{151.66x10^6}}$$

$= 0.95$

$$\lambda_{LT2} = \sqrt{\frac{1.2x488.9x10^3 x250}{151.66x10^6}} = 0.98$$

$\therefore\ \lambda_{LT} = 0.95$

$\phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$

$= 0.5[1 + 0.21(0.95 - 0.2) + 0.95^2] = 1.03$

Bending stress reduction factor, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT} + [\phi_{LT}^2 - \lambda_{LT}^2]^{0.5}\right\}}$

$$= \frac{1}{\left\{1.03 + [1.03^2 - 0.95^2]^{0.5}\right\}} = 0.70 \le 1.0$$

Design bending compressive stress, $f_{bd} = \dfrac{\chi_{LT} f_y}{\gamma_{mo}}$

$$= \frac{0.70x250}{1.10} = 159.10$$

Design bending strength, $M_d = \beta_b Z_{pz} f_{bd}$

$$= 1.0x554.32x10^3 x159.10 = 88.22\ kN - m$$

Method-3

Extreme fiber bending compressive stress, $f_{cr,b} = \dfrac{1.1\pi^2 E}{\left(L_{LT}/r_y\right)^2}\left[1 + \dfrac{1}{20}\left(\dfrac{L_{LT}/r_y}{h_f/t_f}\right)^2\right]^{0.5}$

$$= \frac{1.1x\pi^2 x2x10^5}{\left(3000/28\right)^2}\left[1 + \frac{1}{20}\left(\frac{3000/28}{290.6/9.4}\right)^2\right]^{0.5}$$

$= 239.27\ N/mm^2$

Non-dimensional slenderness ratio, $\lambda_{LT} = \sqrt{f_y/f_{cr,b}}$

$$= \sqrt{250/239.27} = 1.022$$

$\phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$

$= 0.5\ [1 + 0.21(1.022 - 0.2) + (1.022)^2] = 1.108$

Bending stress reduction factor, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT} + [\phi_{LT}^2 - \lambda_{LT}^2]^{0.5}\right\}}$

$$= \frac{1}{\left\{1.108 + [1.108^2 - 1.022^2]^{0.5}\right\}}$$

$= 0.65 \le 1.0$

Design bending compressive stress, $f_{bd} = \dfrac{\chi_{LT} f_y}{\gamma_{mo}}$

$$= \frac{0.65x250}{1.10} = 147.96$$

Design bending strength, $M_d = \beta_b Z_p f_{bd}$

$$= 1.0x554.32x10^3 x147.96 = 82.02\ kN - m$$

Method-4

$$\frac{kL}{r} = \frac{3000}{28} = 107.142$$

$$h/t_f = \frac{300}{9.4} = 31.914$$

Form Table 14 of IS: 800-2007, $f_{cr,b}$ is calculated by double interpolation

kL/r	h/t_f		
	30	**31.91**	35
100	270.9	*235.847*	257.7
107.14		***237.730***	
110	232.1	*227.200*	219.3

$\therefore$ Critical stress $f_{cr,b} = 237.730\ N/mm^2$

Design bending compressive stress corresponding to lateral buckling (*From Table 13 of IS: 800-2007)*, $f_{bd} = 147.833\ N/mm^2$

Design bending strength, $M_d = \beta_b z_p f_{bd}$

$$= 1.0x554.32x10^3 x147.833 = 81.94\ kN\text{-}m$$

Working stress method

From Clause 6.2.2 of IS: 800-1984

Permissible bending compressive stress, σ_{bc}

$d_1 = 300 - 2(9.4) = 281.2\ mm$

Mean thickness of compression flange, T = 9.4 *mm*

Web thickness, t = 6.4 *mm*

Overall depth of web, D = 300 *mm*

$D/T = 300/9.4 = 31.91$

$T/t = 9.4/6.7 = 1.40 \le 2.0$

$d_1/t = 281.2/6.7 = 41.97 \le 85$

Effective length of compression flange, l_{eff} = 1.0x3000 = 3000 *mm*

$l/r_y = 3000/28 = 107.14$

From Table 6.1B of IS: 800-1984, σ_{bc} is calculated by double interpolation

kL/r	D/T		
	30	**31.91**	35
100	122	*121.236*	120
107.14		***116.238***	
110	115	*114.236*	113

∴ Bending compressive stress, σ_{bc} = 116.24 *N/mm²*

Design bending moment of section, M_r = 116.24x488.9x10³ = 56.83 *kN-m*

Considering the section ISHB 300 at 63.0 *kg/m*

Sectional properties:

Sectional area, A = 8025 mm^2

Depth of section, D = 300 *mm*

Width of flange, b_f = 250 *mm*

Moment of inertia, I_{zz} = 12950.6 cm^4

Moment of inertia, I_{yy} =2246.7 cm^4

Plastic modulus, Z_{pz} = 962.18 cm^3

Radius at root, r_1= 11.0 *mm*

Thickness of flange, t_f = 10.6 *mm*

Thickness of web, t_w = 9.4 *mm*

Radius of gyration, r_{zz} = 12.70 *cm*

Radius of gyration, r_{yy} = 5.29 *cm*

Section modulus, Z_{ez} = 863.3 cm^3

Section modulus, Z_{ey} = 178.4 cm^3

Depth of web, d = 249.8 *mm*

d_2 = 25.1 *mm*

Section classification *Table-2 of code IS: 800-2007*

$$\varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Outstanding flanges:

$$\frac{b}{t_f} = \frac{250}{2x10.6} = 11.79 < 15.7\varepsilon$$

Hence flange is semi-compact element.

Web of an I-section:

$$\frac{d}{t_w} = \frac{249.8}{9.4} = 26.57 \; < \; 84\varepsilon$$

Hence web is a plastic element.

Hence section is considered as semi-compact section.

Method-1

To determine design bending moment of the section:

Elastic lateral buckling, $M_{cr} = \sqrt{\left[\left(\frac{\pi^2 EI_y}{(L_{LT})^2}\right)\left(GI_t + \frac{\pi^2 EI_w}{(L_{LT})^2}\right)\right]}$

Torsional constant, $I_t = \sum b_i t_i^3/3$

$$= \left[\left(\frac{2x250x10.6^3}{3}\right) + \left(\frac{289.4x9.4^3}{3}\right)\right] = 278.626x10^3 \, mm^4$$

Warping constant, $I_w = (1-\beta_f)\beta_f I_y h_f^2$

$$I_{fc} = I_{ft} = \frac{10.6x250^3}{12} = 13.80x10^6 \; mm^4$$

$$\beta_f = \frac{I_{fc}}{I_{fc}+I_{ft}} = \frac{13.80x10^6}{(13.80x10^6 + 13.80x10^6)} = 0.50$$

$h_f = (D-t_f) = (300-10.6) = 289.4$ *mm*

$\therefore I_w = (1-0.5)x0.5x2246.7x10^4 x(289.4)^2 = 4.704x10^{11}$ mm^4

$$G = \frac{E}{2(1+\mu)} = \frac{2x10^5}{2(1+0.3)} = 76.923x10^3 \; N/mm^2$$

Substituting (a), (b), (c), (d) in equation (2.7)

$$M_{cr} = \sqrt{\left[\left(\frac{\pi^2 x2x10^5 x2246.7x10^4}{(3000)^2}\right)\left((76.923x10^3 x278.626x10^3) + \left(\frac{\pi^2 x2x10^5 x4.764x10^{11}}{(3000)^2}\right)\right)\right]}$$

= 783.574 *kN-m*

$$\lambda_{LT1} = \sqrt{\frac{\beta_b Z_{pz} f_y}{M_{cr}}} \le \lambda_{LT2} = \sqrt{\frac{1.2 Z_{ez} f_y}{M_{cr}}}$$

$$\lambda_{LT1} = \sqrt{\frac{0.897x962.18x10^3 x250}{783.574x10^6}} = 0.525$$

$$\lambda_{LT2} = \sqrt{\frac{1.2x863.3x10^3 x250}{783.574x10^6}} = 0.575$$

$\therefore\ \lambda_{LT} = 0.525$

$$\phi_{LT} = 0.5[1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^2]$$

$$= 0.5[1+0.21(0.525-0.2)+0.525^2] = 0.672$$

Bending stress reduction factor, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT}+[\phi_{LT}^2-\lambda_{LT}^2]^{0.5}\right\}}$

$$= \frac{1}{\left\{0.672+[0.672^2-0.525^2]^{0.5}\right\}} = 0.916 \le 1.0$$

Design bending compressive stress, $f_{bd} = \dfrac{\chi_{LT} f_y}{\gamma_{mo}}$

$$= \frac{0.916\text{x}250}{1.10} = 208.18$$

Design bending strength, $M_d = \beta_b Z_p f_{bd}$

$$= 0.897\text{x}962.18\text{x}10^3\text{x}208.18 = 179.675\ kN-m$$

Since the section is semi-compact $\beta_b = (Z_{pz}/Z_{ez})$ $\therefore \beta_b = (863.3/962.18) = 0.897$

Method-2

Using the simplified equation which can be used for standard rolled I- section

Elastic lateral buckling moment, $M_{cr} = \dfrac{\pi^2 EI_y h_f}{2L_{LT}^2}\left[1+\dfrac{1}{20}\left(\dfrac{L_{LT}/r_{yy}}{h_f/t_f}\right)^2\right]^{0.5}$

Assuming the member is torsional fully restraint & warping not restrained in both flanges *(from Table-2 of code IS: 800-2007)*

Effective length for lateral torsional buckling, L_{LT} =1.0xL = 1.0x3.0

=3.0 *m*

Center-to-center distance between flange, $h_f = (D - t_f)$

= (300 – 10.6) = 289.4 *mm*

$$M_{cr} = \frac{\pi^2\text{x}2\text{x}10^5\text{x}2246.7\text{x}10^4\text{x}289.4}{2\text{x}3000^2}\left[1+\frac{1}{20}\left(\frac{3000/52.9}{289.4/10.6}\right)^2\right]^{0.5}$$

= 786.174*kN-m*

$$\lambda_{LT1} = \sqrt{\frac{\beta_b Z_{pz} f_y}{M_{cr}}} \le \lambda_{LT2} = \sqrt{\frac{1.2 Z_{ez} f_y}{M_{cr}}}$$

$$\lambda_{LT1} = \sqrt{\frac{0.897x962.18x10^3x250}{786.174x10^6}} = 0.523$$

$$\lambda_{LT2} = \sqrt{\frac{1.2x863.3x10^3x250}{786.174x10^6}} = 0.574$$

$\therefore\ \lambda_{LT} = 0.523$

$$\phi_{LT} = 0.5[1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^2]$$

$$= 0.5[1+0.21(0.523-0.2)+0.523^2] = 0.67$$

Bending stress reduction factor, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT}+[\phi_{LT}^2-\lambda_{LT}^2]^{0.5}\right\}}$

$$= \frac{1}{\left\{0.67+[0.67^2-0.523^2]^{0.5}\right\}} = 0.918 \le 1.0$$

Design bending compressive stress, $f_{bd} = \dfrac{\chi_{LT} f_y}{\gamma_{mo}}$

$$= \frac{0.917x250}{1.10} = 208.636$$

Design bending strength, $M_d = \beta_b Z_p f_{bd}$

$$= 0.897x962.18x10^3x208.636 = 180.068\ kN-m$$

Method-3

Extreme fiber bending compressive stress, $f_{cr,b} = \dfrac{1.1\pi^2 E}{\left(L_{LT}/r_y\right)^2}\left[1+\dfrac{1}{20}\left(\dfrac{L_{LT}/r_y}{h_f/t_f}\right)^2\right]^{0.5}$

$$= \frac{1.1x\pi^2x2x10^5}{\left(3000/52.9\right)^2}\left[1+\frac{1}{20}\left(\frac{3000/52.9}{289.9/10.6}\right)^2\right]^{0.5}$$

$= 744.406\ N/mm^2$

Non-dimensional slenderness ratio, $\lambda_{LT} = \sqrt{f_y/f_{cr,b}}$

$$= \sqrt{250/744.406} = 0.579$$

$$\phi_{LT} = 0.5[1+\alpha_{LT}(\lambda_{LT}-0.2)+\lambda_{LT}^2]$$

$$= 0.5\ [1+0.21(0.579-0.2)+(0.579)^2] = 0.708$$

Bending stress reduction factor, $\chi_{LT} = \dfrac{1}{\left\{\phi_{LT}+[\phi_{LT}^2-\lambda_{LT}^2]^{0.5}\right\}}$

$$= \frac{1}{\left\{0.708 + [0.708^2 - 0.579^2]^{0.5}\right\}} = 0.897 \leq 1.0$$

Design bending compressive stress, $f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{mo}}, \ = \frac{0.897 x 250}{1.10} = 204.00 \ \text{MPa}$

Design bending strength, $M_d = \beta_b Z_p f_{bd}$

$$= 0.897 x 962.18 x 10^3 x 204 = 176.067 \ kN - m$$

Method-4

$\frac{kL}{r_y} = \frac{3000}{52.9} = 56.71$, $h/t_f = \frac{300}{10.6} = 28.30$, Form Table 14 of IS: 800-2007, $f_{cr,b}$ is calculated by double interpolation

kL/r	h/t_f		
	25	**28.30**	30
50	951.7	*935.46*	927.1
56.71		**756.637**	
60	684.6	*668.958*	660.9

$\therefore$ Critical stress $f_{cr,b} = 756.637$ *N/mm²*

Design bending compressive stress corresponding to lateral buckling (*From Table 13 of IS: 800-2007)*, $f_{bd} = 204.848$ *N/mm²*

Design bending strength, $M_d = \beta_b z_p f_{bd}$

$$= 0.897 x 554.32 x 10^3 x 204.848 = 101.85 \ kN\text{-}m$$

Working stress method

From Clause 6.2.2 of IS: 800-1984

Permissible bending compressive stress, σ_{bc}

$d_1 = 300 - 2(10.6) = 278.8$ *mm*

Mean thickness of compression flange, T = 10.6 *mm*

Web thickness, t = 9.4 *mm*

Overall depth of web, D = 300 *mm*

$D/T = 300/10.6 = 28.30$

$T/t = 10.6/9.4 = 1.13 \leq 2.0$

$d_1/t = 278.8/9.4 = 29.66 \leq 85$

Effective length of compression flange, l_{eff} = 1.0x3000 = 3000 *mm*

$l/r_y = 3000/52.9 = 56.71$

From Table 6.1B of IS: 800-1984

kL/r	D/T		
	25	**28.30**	30
55	153	*152.34*	152
56.71		***151.314***	
60	150	*149.34*	149

∴ Bending compressive stress, σ_{bc} = 151.314 N/mm^2

Moment of resistance, M = 151.314x863.3x10^3 =130.63 *kN-m*

Other types of problems

(6) A hall 8m x 12 m is provided with a100 mm RCC slab over RSJ spaced 3m c/c. A wearing load of 100mm is provided over the slab. The live load on the beam is 1.5 kN/ m^2. Design the beam (RSJ) if it is laterally supported.

Solution: Find the total load on the beam and then BM and SF

Dead load due to self weight and wearing load

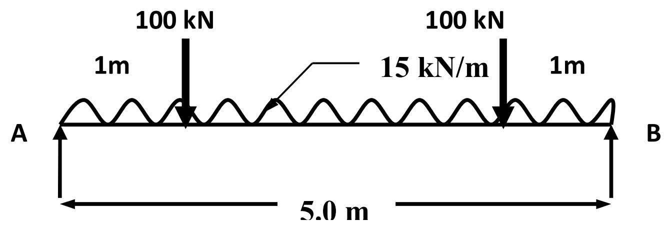
= [(100 + 100)/ (1000)] x 24 x 3 x1 kN/m =14.4 kN/m

Live load = 1.5 kN/m, Self weight of joint =0.5 kN/m, Total load =16.4 kN/m

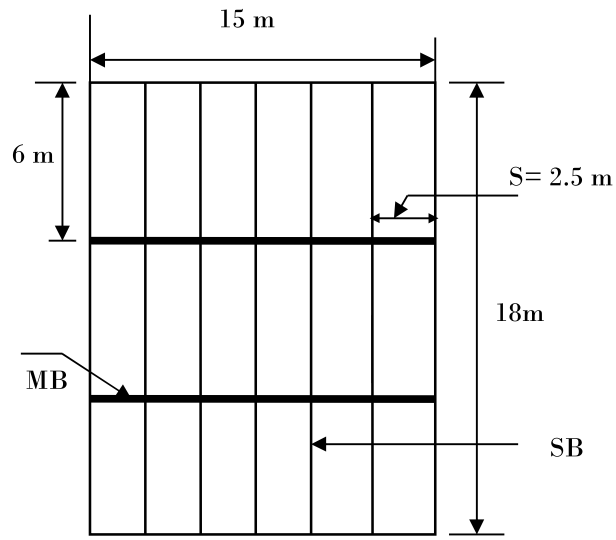
Therefore total factored load w = 1.5 x 16.4 =24.6 kN/m

Rest of the design is as usual.

(7) Design the beam shown in the figure. Use Fe410 steel. The beam is assumed to be laterally supported.



(8) A hall in a building 15m x 18 m is supporting a slab 150mm thick and is shown in figure. Design both secondary and primary beams. Use Fe410 steel. The density of concrete is 24kN/m^3. The imposed load on the beam is 4 kN/m^2. Self weight of beam may be assumed as 0.5 kN/m^2.



(9) Determine the plastic section modulus of rectangular section, triangular section, T, I, O, [and other sections.

(10) Determine the following properties of the sections such as I, T, [.
I_{zz}, I_{yy}, Z_{ez}, Z_{pz}, shape factor and M_p

(11) Plastic analysis of beams: Fixed, cantilever, propped cantilever and continuous beams for different loads. Analyse for Plastic moment and design by LSM. Uniform sections or varying sections can be designed. Once Mp is found, select a suitable section so to have the required Mp.

(12) Portal frames for varying loads can be worked out for analysis and design.

Similarly workout other problems related to built up sections and plated sections.

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