1 Explain the types of fluid flow

→ The fluid flow is classified as

* Steady and unsteady flow
* Uniform and Non-uniform flow
* Laminar and turbulent flow
* Compressible and incompressible flow
* Rotational and irrotational flow
* one, two and three dimensional flow

• <u>Steady and unsteady flow</u>

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density and etc at a point do not change with time. Thus, for steady flow mathematically we have

$$\left(\frac{\partial v}{\partial t}\right)_{(x_0, y_0, z_0)} = 0 \quad , \quad \left(\frac{\partial p}{\partial t}\right)_{(x_0, y_0, z_0)} = 0 \quad , \quad \left(\frac{\partial \rho}{\partial t}\right)_{(x_0, y_0, z_0)} = 0$$

where, (x_0, y_0, z_0) is fixed at a point in the fluid feild.

unsteady flow is that type of flow in which the velocity, pressure, density at a point changes with respect to time. Thus, mathematically for unsteady flow

$$\left(\frac{\partial v}{\partial t}\right)_{(x_0, y_0, z_0)} \neq 0 \quad , \quad \left(\frac{\partial p}{\partial t}\right)_{(x_0, y_0, z_0)} \neq 0$$

- **Uniform and Non-uniform flow**

Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (i.e. direction of flow). Mathematically, for uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{constant}} = 0$$

where ∂v = change of velocity
∂s = Length of flow in direction s

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect in space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{constant}} \neq 0$$

- **Laminar and Turbulent flow**

Laminar flow is defined as that type of flow in which the fluid particles move along the well defined path or streamline and all the streamlines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layers. This type of flow is called stream line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way. Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynolds number.

where, D - diameter of the pipe

V - Mean velocity of the flow in pipe

ν - Kinematic viscosity of the fluid or ν

If the Reynolds number is less than 2000, the flow is laminar. If it is between 2000 and 4000, it means the flow may be laminar or turbulent, if it is more than 4000, it is called turbulent flow

- **Compressible and Incompressible flow**

Compressible flow is the flow in which density of the flow changes from point to point or in other the density (ρ) is not constant. Thus mathematically for compressible flow

$$\rho \neq \text{constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible whi gases are compressible. Mathematically for incompressible flow,

$$\rho = \text{constant}.$$

- **Rotational and Irrotational flow**

Rotational flow is that type of flow in which the fluid particles while flowing along stream lines, also rotate about their own axis.

If the fluid particles while flowing, along stream-lines do not rotate about their own it is called irrotational flow.

- **One, two and three dimensional flow**

One dimensional flow is that type of flow in which the flow parameters such as velocity is a function of time and one space coordinate only, say x. For a steady one dimensional the velocity is a function of one space-coordinate only. The variation of velocities in the other mutually perpendicular directions is assumed negligible. Hence mathematically, for one-dimensional flow

$$u = f(x), \quad v = 0 \quad \text{and} \quad w = 0$$

where, u, v and w are velocity components in x, y and z directions respectively.

Two dimensional flow is that type of flow in which the velocity is a function of time and two space or rectangular coordinate say x, y. For a steady two dimensional flow, the variation of velocity in the third dimension is negligible. Thus mathematically for a two dimensional flow

$$u = f_1(x, y), \quad v = f_2(x, y), \quad w = 0$$

Three dimensional flow is the type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three dimensional flow, the fluid parameters are functions of three space coordinates (x, y, z) only. Thus mathematically, for a three dimensional flow

$$u = f_1(x, y, z) \quad v = f_2(x, y, z) \quad w = f_3(x, y, z)$$

2. The diameters of a pipe at a sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of the water flowing through the pipe at section 1 is $5\,m\,s^{-1}$. Determine the velocity at section 2.

Solution:

Given; At section 1

$D_1 = 10\,cm = 0.1\,m$

$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(0.1)^2$

$A_1 = 0.007854\,m^2$

$V_1 = 5\,m\,s^{-1}$.

At section 2

$D_2 = 15\,cm = 0.15\,m$

$A_2 = \frac{\pi}{4}(D_2^2) = \frac{\pi}{4}(0.15)^2$

$A_2 = 0.01767\,m^2$

$V_2 = ?$

1) Discharge through pipe is given by

$Q = A_1 \times V_1$

$= 0.007854 \times 5$

$Q = 0.03927\,m^3/s$

we have,

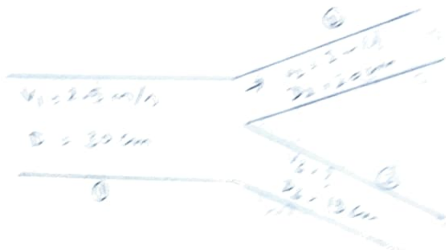
$A_1 V_1 = A_2 V_2$

$V_2 = \frac{A_1 V_1}{A_2}$

$= \frac{0.007854 \times 5}{0.01767}$

$\boxed{V_2 = 2.22\,m/s}$

2. A 30 cm diameter pipe, conveying water branches into two pipes of diameter 20 cm and 15 cm respectively. If the average velocity of the 30 cm diameter pipe is 2.5 m/s find the discharge in velocity in the 15 cm pipe if the average velocity of 20 cm pipe is 2 m/s.



→ Solution :

Given - $D_1 = 30\ cm = 0.3m$

$A_1 = \frac{\pi}{4}(D_1)^2 = \frac{\pi}{4}(0.3)^2 = 0.07068\ m^2$

$V_1 = 2.5\ m/s$

$D_2 = 20\ cm = 0.2\ m$

$A_2 = \frac{\pi}{4}(0.2)^2 = 0.0314\ m^2$

$V_2 = 2\ m/s$

$D_3 = 15\ cm = 0.15\ m$

$A_3 = \frac{\pi}{4}(0.15)^2 = 0.01767\ m^2$

$V_3 = ?$

To find ① Discharge in pipe 1 & 3

② velocity in pipe of diameter 15 cm i.e. V_3

Let Q_1, Q_2 and Q_3 be the discharge in pipe 1, 2 and 3 respectively

Then according to the continuity equation,

$Q_1 = Q_2 + Q_3$ — ①

① The discharge Q_1 in pipe 1 is given by

$$Q_1 = A_1 V_1$$
$$= 0.07068 \times 2.5$$
$$Q_1 = 0.1767 \ m^3/s$$

② value of V_3

$$Q_2 = A_2 V_2$$
$$= 0.0314 \times 2$$
$$Q_2 = 0.0628 \ m^3/s$$

substituting the values of Q_1 and Q_2 in equation ①

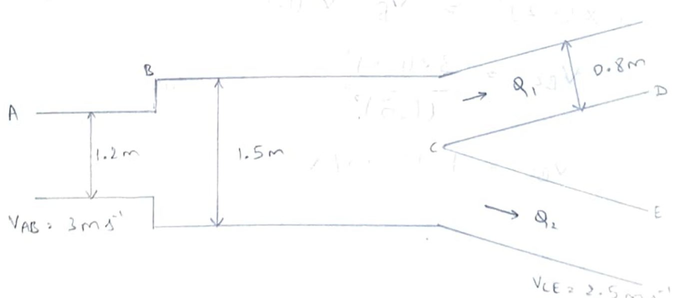
$$0.1767 = 0.0628 + Q_3$$
$$Q_3 = 0.1139 \ m^3/s$$

$$Q_3 = A_3 V_3$$

$$V_3 = \frac{Q_3}{A_3} = \frac{0.1139}{0.01767}$$

$$\boxed{V_3 = 6.44 \ m/s}$$

4. water flows through a pipe AB of diameter 1.2 m at $3 \ m s^{-1}$ and then passes through a pipe BC of diameter 1.5m. At C, the pipe branches, branched CD is 0.8 m in diameter and carries one third of the flow in AB. The flow velocity in branch CE is $2.5 \ m s^{-1}$. Find the volume rate of flow in AB, the velocity in BC, that in CD and the diameter of CE.



→ Solution :

Given - Diameter of pipe AB, $D_{AB} = 1.2\ m$

velocity of flow through AB, $V_{AB} = 3\ m s^{-1}$

Diameter of pipe BC, $D_{BC} = 1.5 m$

Diameter of branched pipe CD, $D_{CD} = 0.8\ m$

velocity of flow in pipe CE $= V_{CE} = 2.5\ m/s$

Let the flow rate in pipe AB $= Q\ m^3/s$

Let the velocity of flow in pipe BC $= V_{BC} = V_{BC}\ m/s$

velocity of flow in pipe CD $= V_{CD}\ m/s$

Diameter of pipe CE $= D_{CE}$

Then, flow rate through CD $= Q/3$

and flow rate through CE $= Q - Q/3 = \frac{2Q}{3}$

(i) Now volume flow rate through AB $= Q$

$= V_{AB} \times$ Area of AB

$= 3 \times \frac{\pi}{4} (D_{AB})^2$

$= 3 \times \frac{\pi}{4} (1.2)^2$

$= 3.393\ m^3/s$

(ii) Applying continuity equation to pipe AB and pipe BC

$V_{AB} \times$ Area of pipe AB $= V_{BC} \times$ Area of pipe BC

$$3 \times \frac{\pi}{4} (D_{AB})^2 = V_{BC} \times \frac{\pi}{4} (D_{BC})^2$$

$$3 \times (1.2)^2 = V_{BC} \times (1.5)^2$$

$$V_{BC} = \frac{3 \times (1.2)^2}{(1.5)^2}$$

$$\underline{V_{BC} = 1.92\ m/s}$$

(iii) The flow rate through pipe CD

$CD = Q_1 = Q/3 = \frac{3.393}{3} = 1.131 \ m^3/s$

$Q_1 = V_{CD} \times$ Area of pipe CD

$1.131 = V_{CD} \times \frac{\pi}{4} (D_{CD})^2$

$1.131 = V_{CD} \times \frac{\pi}{4} (0.8)^2$

$V_{CD} = \frac{1.131}{0.5026}$

$\underline{V_{CD} = 2.25 \ m/s}$

(iv) Flow rate through CE

$Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262 \ m^3/s$

$Q_2 = V_{CE} \times$ Area of pipe CE

$Q_2 = V_{CE} \times \frac{\pi}{4} (D_{CE})^2$

$2.262 = V_{CE} \times \frac{\pi}{4} (D_{CE})^2$

$2.262 = 2.5 \times \frac{\pi}{4} (D_{CE})^2$

$D_{CE} = \sqrt{\frac{2.262 \times 4}{2.5 \times \pi}}$

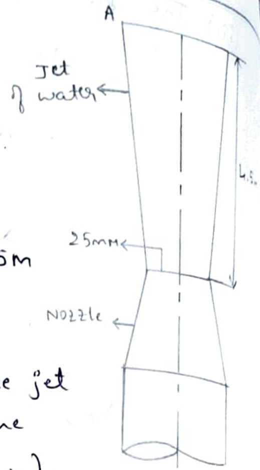
$D_{CE} = 1.0735 \ m$

Diameter of pipe CE = $\underline{1.0735 \ m}$

5. A jet of water from a 25 mm diameter nozzle is directed vertically upwards. Assuming that the jet remain circular and neglecting any loss of energy that will be the diameter at a point 4.5 m above the nozzle, if the velocity with the jet leaves the nozzle is $12 \ ms^{-1}$

Solution:

Given - Diameter of nozzle, $D_1 = 25\,mm$
$= 0.025\,m$



velocity of jet at nozzle $= V_1 = 12\,m s^{-1}$.

Height of point A, $h = 4.5\,m$

Let the velocity of the jet at height 4.5m be V_2

Consider the vertical motion of the jet from the outlet of the nozzle to the point A (neglecting any loss of energy)

Initial velocity, $u = V_1 = 12\,m s^{-1}$.

Final velocity, $v = V_2 = ?$

value of $g = 9.81\,m s^{-2}$.

$h = 4.5\,m$

using $v^2 - u^2 = 2gh$, we get

$V_2^2 - V_1^2 = 2gh$

$V_2^2 - (12)^2 = 2 \times (-9.81) \times 4.5$

$V_2 = \sqrt{12^2 - 2 \times (-9.81) \times 4.5}$

$V_2 = 7.46\,m/s$

Now applying continuity equation to the outlet of nozzle and at point A, we get

$$A_2 = \frac{A_1 V_1}{V_2} = \frac{\pi/4\, D_1^2\, V_1}{V_2}$$

$$A_2 = \frac{\pi \times (0.025)^2 \times 12}{4 \times 7.46}$$

$$A_2 = 0.0007896\,m^2$$

Let D_2 = Diameter of jet at point A

Then $A_2 = \frac{\pi}{4} D_2^2$

$$0.0007896 = \frac{\pi}{4} D_2^2$$

$$D_2 = \sqrt{\frac{0.0007896 \times 4}{\pi}}$$

$$\underline{D_2 = 0.0317\,m}$$

$\therefore$ The diameter of jet at point A is <u>31.7 mm</u>

6. **Discuss the fluid properties**

a) **Density or mass density**

Density or mass density of a fluid is defined as the ratio of mass of a fluid to its volume. Thus, mass per unit volume of a fluid is called density. It is denoted by the symbol ρ (rho). The unit of mass density in SI units is Kg/m^3.

The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

Mathematically, mass density is written as

$$\text{Density} = \frac{\text{Mass of the fluid}}{\text{volume of the fluid}} = \frac{m}{v} = \rho = Kg/m^3$$

The value of density of water is $1\,gm/cm^3$ or $1000\,Kg/m^3$

b) **Specific weight or weight Density**

Specific weight or weight density of a fluid is the ratio between the weight of the fluid to its volume. Thus, the weight per unit volume of a fluid is called weight density of a fluid and it is denoted as 'w'.

Thus, mathematically

$$w = \frac{\text{weight of fluid}}{\text{Volume of fluid}} = \frac{\text{mass of fluid} \times \text{acceleration due to gravity}}{\text{volume of fluid}}$$

$$= \frac{\text{mass of fluid} \times g}{\text{Volume of fluid}} \qquad \left[\because \frac{\text{mass of fluid}}{\text{volume of fluid}} \right.$$

$$w = \rho \times g$$

The value of specific weights or weight density for water is $9.81 \times 1000 \ N/m^3$ in SI units.

c) **Specific volume**

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass of fluid or volume per unit. Unit mass of a fluid is called specific volume.

Mathematically it is expressed as

$$\text{Specific volume} = \frac{\text{volume of fluid}}{\text{mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{volume of fluid}}}$$

$$\therefore \text{specific volume} = \frac{1}{\rho}$$

Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg. It is commonly applied to gas.

d) **Specific gravity**

It is defined as the ratio of weight of fluid to the weight density of water or the density of fluid to density of water.

Hence mathematically,

$$\text{Specific gravity (S) for liquids} = \frac{\text{Density of liquid}}{\text{Density of water}}$$

$$\text{Specific gravity for gases} = \frac{\text{Density of gas}}{\text{Density of air}}$$

Thus, weight density of a liquid $= S \times$ density of water

$= S \times 9.81 \times 1000 \ N/m^3$

Thus, Density of a liquid $= S \times$ density of water

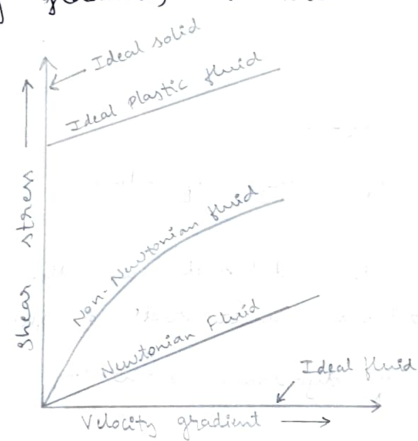
$= S \times 1000 \ Kg/m^3$

If the specific gravity of a fluid is known, then the density of fluid will be equal to the specific gravity of the fluid multiplied by the density of water.

7. Define Newtonian and Non-Newtonian fluid

→ **Newtonian fluid:**

A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient) is known as a Newtonian fluid



Types of Fluids

Non-Newtonian fluid:

A real fluid in which the shear stress is proportional to the rate of shear strain (or velocity gradient) known as a Non-Newtonian fluid.

8 Define Kinematic viscosity and Dynamic viscosity

→ **Kinematic Viscosity:**

It is defined as the ratio between the dynamic viscosity and the density of the fluid. It is denoted by 'nue' greek symbol ν.

Mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

SI unit of kinematic viscosity is m^2/s or cm^2/s

One stoke $= cm^2/s = \left(\frac{1}{100}\right)^2 m^2/s = 10^{-4}\, m^2/s$

Dynamic Viscosity:

It is defined as the variation of shear stress to the rate of shear strain or the rate of shear deformation of velocity gradient. It is denoted by the symbol μ.

Mathematically,

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

where, τ = shear stress

$\frac{du}{dy}$ = rate of change of velocity

It is also defined as the shear stress required to produce unit rate of shear strain.

The SI unit of dynamic viscosity is Ns/m^2

9 What do you understand by Adiabatic and Isothermal process?

→ <u>Isothermal Process</u>:

If the changes in density occurs at constant temperature, then the process is called isothermal process and the relationship between pressure (p) and density (ρ) is given by

$$\frac{p}{\rho} = \text{constant}.$$

<u>Adiabatic Process</u>:

If the change in density occurs with no heat exchange to and from the gas, the process is called Adiabatic and if no is generated within the gas due to friction, the relationship between pressure and density is given by

$$\frac{p}{\rho^k} = \text{constant}$$

Where, k = ratio of specific heat of a gas at constant pressure and constant volume.

10 What is surface Tension? Prove that surface tension for a fluid drop is $\frac{4\sigma}{d}$

→ <u>Surface Tension</u>:

It is defined as the tensile force acting on the surface of a liquid in contact with a gas or the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

Surface Tension on liquid droplet :

Consider a small spherical droplet of a liquid of radius 'r'. On the entire surface of the droplet the tensile force due to surface tension will be acting

Let, σ = Surface tension of the liquid

p = pressure intensity inside the droplet (in excess of the outside pressure intensity)

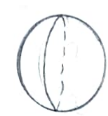
d = diameter of the droplet.

Let the droplet be cut into two halves. The f... acting on one half (say left half) will be

(i) Tensile force due to surface tension acting around the circumference of the cut portion as shown and this is equal to

$= \sigma \times \text{circumference}$

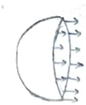
$= \sigma \times \pi d$



Droplet

(ii) Pressure force on the area $\frac{\pi}{4} d^2$

$= p \times \frac{\pi}{4} d^2$



Surface Tension

These two forces will be equal and opposite under equilibrium conditions i.e,

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2}$$

$$\boxed{p = \frac{4\sigma}{d}}$$

Above equation shows that with the decrease of diamet...

of the droplet, pressure intensity inside the droplet increases

11 Derive an expression for force exerted on a vertical plane submerged in liquid.

Consider a plane vertical surface of arbitrary shape immersed in a liquid.

Let, A = Total area of the surface

$\bar{h}$ = Distance of C.G of the area from the free surface of the liquid

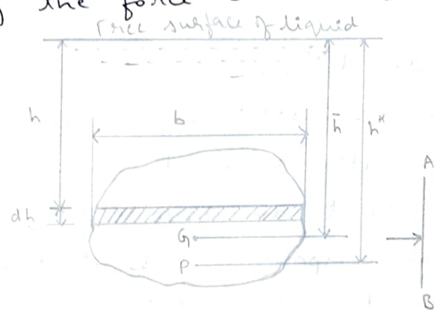
G = Centre of gravity of plane surface

P = Centre of pressure

h^* = distance of centre of pressure from free surface of liquid.

a) Total Pressure (F):

The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated and by integrating the force on a small strip.



Consider a strip of thickness dh and width 'b' at a depth of h from the free surface of liquid.

Pressure intensity on the strip,

$$P = \rho g h$$

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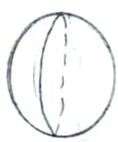

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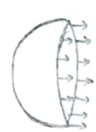

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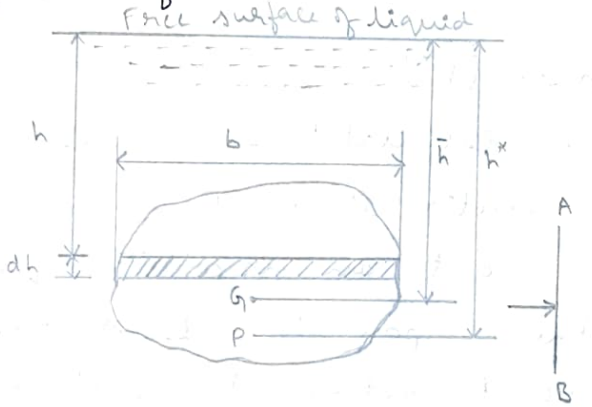


Consider a strip of thickness dh and width 'b' at a depth of h from the free surface of liquid.

Pressure intensity on the strip,

$$P = \rho g h$$

Area of the strip

$dA = b \times dh$

Total pressure force on the strip = $dF = p \times$ Area

$= \rho g h \times b \times dh$

$\therefore$ The total pressure force on the whole surface

$F = \int dF$

$= \int \rho g h \times b \times dh$

$= \rho g \int b \times h \times dh$

but $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × distance of C.G from the free surface

$= A \times \bar{h}$

$F = \rho g A \bar{h}$

For water, the value of $\rho = 1000\ kg/m^3$ and $g = 9.81\ m/s^2$. The force will be in Newton.

b) Centre of Pressure (h^*):

It is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F acting at P, at a distance h^* from free surface of liquid $= F \times h^*$ —— ①

Hence, moment of the force F about free surface of the liquid $= dF \times h$ $(\because dF = \rho g h \times b \times dh)$

$= \rho g h \times b \times dh \times h$

Sum of moments of all such forces about the free surface of the liquid

$$= \int \rho g h \times b \times dh \times h$$

$$= \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 dh$$

$$= \rho g \int h^2 dA \qquad \left[\because \int h^2 dA = \int b h^2 dh\right]$$
$$\because b\, dh = dA$$

but, $\int h^2 dA = \int b h^2 dh$

= Moment of Inertia of the surface about free surface of liquid

$= I_o$

$\therefore$ Sum of moments about free surface $= \rho g I_o$ — (2)

Equating (1) and (2)

$$F \times h^* = \rho g I_o$$

but, $F = \rho g A \bar{h}$

$$\therefore \rho g A \bar{h} \times h^* = \rho g I_o$$

$$h^* = \frac{\rho g I_o}{\rho g A \bar{h}} = \frac{I_o}{A \bar{h}}$$

By theorem of parallel axis, we have

$$I_o = I_G + A\bar{h}^2 \quad \text{— (3)}$$

where, I_G = Moment of inertia of an area about an axis passing through C.G of the area and parallel to the free surface of the liquid

substituting I_G we get

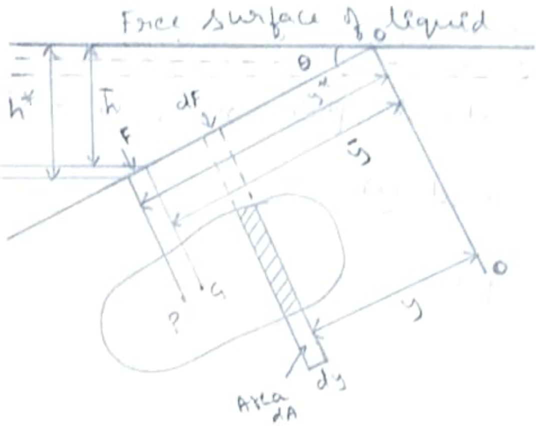
$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

Thus,
$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

$\bar{h}$ is the distance of C.G of the area of the vertical surface from free surface of the liquid

12 Derive an expression for force exerted on inclined plane surface submerged in liquid

→ Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid.



Let, A = Total area of inclined surface
$\bar{h}$ = Depth of C.G of inclined area from free surface
h^* = Distance of centre of pressure from free surface
θ = Angle made by plane of the surface with the free surface of liquid.

Let the plane of the surface, if produced meet the free surface of the liquid at O. Then O-O axis perpendicular to the plane of the surface.

Let, $\bar{y}$ = distance of C.G of inclined surface from O-O
y^* = distance of the centre of pressure from O-O

Consider a small strip of area dA at a depth 'h' from free surface and at a distance 'y' from the axis 0-0.

Pressure intensity on the strip, $p = \rho g h$

$\therefore$ Pressure force dF on strip, $dF = p \times$ Area of strip

$$dF = \rho g h \times dA$$

Total pressure force on the whole area,

$$F = \int dF = \int \rho g h \, dA$$

but from figure, $\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta$

$$\therefore h = y \sin\theta$$

$$F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int y \, dA$$

but, $\int y \, dA = A\bar{y}$

where, $\bar{y}$ = distance of C.G from axis 0-0

$$F = \rho g \sin\theta \, \bar{y} \times A$$

$$F = \rho g A \bar{h} \qquad [\because \bar{h} = \bar{y}\sin\theta]$$

Centre of pressure (h^*)

Pressure force on the strip, $dF = \rho g h \, dA$

$= \rho g y \sin\theta \, dA$

$[\because h = y\sin\theta]$

Moment of the force, dF, about axis 0-0

$= dF \times y$

$= \rho g y \sin\theta \, dA \times y$

$= \rho g \sin\theta \, y^2 \, dA$

Sum of moments of all such forces about 0-0,

$$= \int \rho g \sin\theta \, y^2 \, dA$$

$$= \rho g \sin\theta \int y^2 \, dA$$

but, $\int y^2 dA$ = Moment of inertia of the surface about O

$= I_0$

$\therefore$ Sum of moments of all forces about O-O

$= \rho g \sin\theta \, I_0$

Moment of the total force F, about O-O is also given by $= F \times y^*$

where y^* = Distance of centre of pressure from O-O

Equating the above two equations, we get

$$F \times y^* = \rho g \sin\theta \, I_0$$

$$y^* = \frac{\rho g \sin\theta \, I_0}{F}$$

$$y^* = \frac{h^*}{\sin\theta} \quad , \quad F = \rho g A \bar{h}$$

I_0 by the theorem of parallel axis $= I_G + A\bar{y}^2$

on substitution,

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A \bar{h}} \left[I_G + A\bar{y}^2 \right]$$

$$h^* = \frac{\sin^2\theta}{A\bar{h}} \left[I_G + A\bar{y}^2 \right]$$

but $\frac{\bar{h}}{\bar{y}} = \sin\theta$ or $\bar{y} = \frac{\bar{h}}{\sin\theta}$

$$h^* = \frac{\sin^2\theta}{A\bar{h}} \left[I_G + \frac{A\bar{h}^2}{\sin^2\theta} \right]$$

3

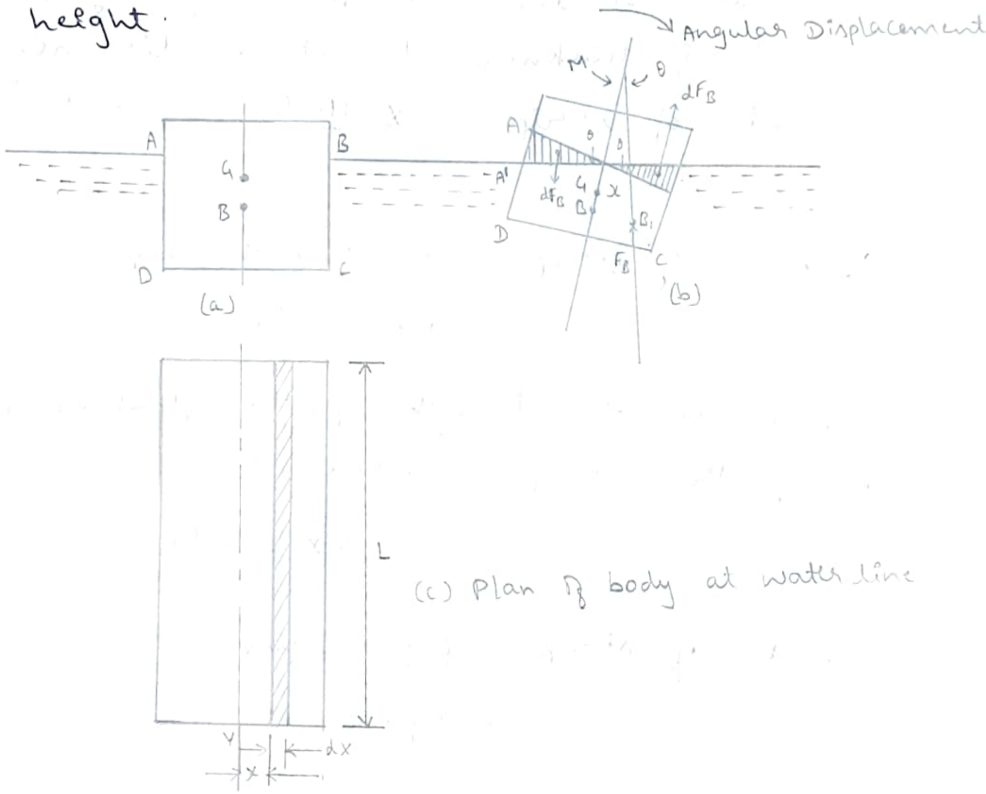
$$\boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$$

If $\theta = 90°$, equation becomes same as previous equation which is applicable to vertically plane submerged surfaces.

I_G = Moment of inertia of inclined surfaces about an axis passing through G and parallel to 0-0 axis.

13 Derive an expression for metacentric height by analytical method.

Figure shows the position of a floating body in equilibrium. The location of centre of gravity and centre of buoyancy in this position is at G and B. The floating body is given a small angular displacement in the clockwise direction. The new centre of buoyancy is at B_1. The vertical line through B_1 cuts the normal axis at M. Hence M is the meta-centre and GM is meta-centric height.



Meta-centric height of a floating body

The angular displacement of the body in the clockwise direction causes the wedge - shaped prism on the right of the axis to go inside the water while the identical wedge-shaped prism represented AOA' emerges out of the water on the left of the axis. These wedges represent a gain in buoyant force on the right side and a corresponding loss of buoyant force on the left side. The gain is represented by a vertical force dF_B acting through the C.G of the prism BOB' while the loss is represented by an equal and opposite force dF_B acting vertically downward through the centroid of AOA'. The couple due to the buoyant forces dF_B tends to rotate the ship in the counter clockwise direction. Also the moment caused by the displacement of the centre of buoyancy from B to ... is also in the counter clockwise direction. Thus these two couples must be equal.

Couple Due to wedges,

Consider towards the right of the axis a small strip of thickness dx at a distance x from O.

The height of strip $x \times \angle BOB' = x \times \theta$ [$\because \angle BOB' = \angle AOA' = \angle BMB_1 = \theta$]

$\therefore$ Area of strip $=$ Height $\times$ thickness

$= x \times \theta \times dx$

If L is the length of the floating body, then volume of the strip $=$ Area $\times L$

$= x \times \theta \times L \times dx$

$\therefore$ weight of strip $= \rho g \times$ volume

$= \rho g x \theta L dx$

Similarly, if a small strip of thickness dx at a distance x from O towards the left of the axis is considered, the weight of strip will be $w \times \theta L\, dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple = weight of each strip × Distance between these two weights

$$= \rho g x \theta L\, dx\, [x + x]$$

$$= \rho g x \theta L\, dx \times 2x$$

$$= 2\rho g x^2 \theta L\, dx$$

∴ Moment of the couple for the whole wedge

$$= \int 2\rho g x^2 \theta L\, dx \quad \text{—} \quad (1)$$

Moment of couple due to shifting of centre of buoyancy from B to B_1

$$= F_B \times BB_1$$

$$= F_B \times BM \times \theta \qquad [\because BB_1 = BM\,\theta, \text{ if } \theta \text{ is very small}]$$

$$= W \times BM \times \theta \quad \text{—} (2) \qquad [\because F_B = W]$$

But these two couples are the same. Hence equating equations (1) and (2), we get

$$W \times BM \times \theta = \int 2\rho g x^2 \theta L\, dx$$

$$W \times BM \times \theta = 2\rho g \theta \int x^2 dx\, L$$

$$W \times BM = 2\rho g \int x^2 L\, dx$$

Now $L\,dx$ = elemental area on the water line and = dA

$$W \times BM = 2\rho g \int x^2 dA$$

But from figure (c) it is clear that $2\int x^2 dA$ is the second moment of area of the plan of the body at water surface about the axis y-y. Therefore

$$W \times BM = \rho g I \qquad \left[I = 2\int x^2 dA\right]$$

$$BM = \frac{\rho g I}{W}$$

But, W = weight of the body

= weight of the fluid displaced by the b.

= ρg × volume of the fluid displaced by body

= ρg × volume of the body sub-merged in water

= $\rho g \times V$

$$BM = \frac{\rho g I}{\rho g V} = \frac{I}{V}$$

$$GM = BM - BG$$

$$\boxed{GM = \frac{I}{V} - BG}$$

∴ Meta-Centric height = $GM = \frac{I}{V} - BG$

14. If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in ms^{-1} at a distance y meters above the plate, determine the shear stress at $y = 0$ and $0.15m$. Take dynamic viscosity of fluid as 8.63 poise.

→ Solution :

Given -

$$u = \frac{2}{3}y - y^2$$

$$\frac{du}{dy} = \frac{2}{3} - 2y$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = 0.6667$$

$$\left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2(0.15) = 0.667 - 0.30 = 0.367$$

value of $\mu = 8.63$ poise, $\frac{8.63}{10}$ SI units $= 0.863$ Ns/m^2

Now, shear stress is given by equation

$$\tau = \mu\left(\frac{du}{dy}\right)$$

(i) Shear stress at $y = 0$ is

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \underline{0.5756\ N/m^2}$$

(ii) Shear stress at $y = 0.15$ is

$$\tau = \mu\left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \underline{0.3167\ N/m^2}$$

---

15. A flat plate of area $1.5 \times 10^6\ mm^2$ is pulled with a speed of $0.4\ ms^{-1}$ relative to another plate located at a distance of 0.15mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity 1 poise.

→ Solution:

Given - Area of the plate, $A = 1.5 \times 10^6\ mm^2 = 1.5\ m^2$

Speed of plate relative to another plate $= 0.4\ ms^{-1}$.

Distance between the plates, $dy = 0.15\ mm = 0.15 \times 10^{-3}\ m$

viscosity $= 1$ poise $= \mu = \frac{1}{10}\ \frac{Ns}{m^2}$

WKT,

$$\tau = \mu\left(\frac{du}{dy}\right)$$

$$= \frac{1}{10}\left(\frac{0.4}{0.15 \times 10^{-3}}\right)$$

$$\tau = 266.66\ N/m^2$$

(i) Shear force

$F = \tau \times \text{area}$

$= 266.66 \times 1.5$

$\underline{F = 400N}$

(ii) Power required to move the plate at that speed (0.4 m/s)

$P = F \times u = 400 \times 0.4$

$\underline{P = 160W}$

16. Calculate the dynamic viscosity of an oil which is for lubrication between a square plate of size 0.8m and an inclined plane with angle 30°. The weight of the square plate is 300N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. the thickness of oil film is 1.5 mm

→ Solution:

Given - Area of plate = $A = 0.8m \times 0.8m = 0.64 m^2$

Angle of plate = $\theta = 30°$

weight of plate = $W = 300N$

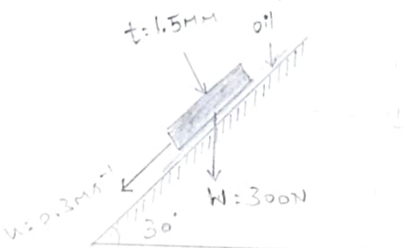
Thickness of oil film, $t = dy = 1.5mm = 1.5 \times 10^{-3} m$

Velocity of plate, $u = 0.3 m/s$

Let the viscosity of fluid between plate and inclined plane is μ

Thus force of shear F, on the bottom surface of the plate = 150N

Shear stress = $\tau = \dfrac{F}{\text{Area}} = \dfrac{150}{0.64}$ N/m^2



WKT,

$$\tau = \mu \frac{du}{dy}$$

where, du = change in velocity , u - 0 = u = 0.3 m/s

dy = t = 1.5×10^{-3} m

$$\tau = \mu \frac{du}{dy}$$

$$\frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3}$$

$$\mu = 1.17 \ Ns/m^2$$

$$\mu = 1.17 \times 10$$

$$\boxed{\mu = 11.7 \text{ poise}}$$

17. The space between two flat square parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 m/s requires a force of 98.1 N to maintain the speed. Find,

(i) The dynamic viscosity of the oil in poise

(ii) The kinematic viscosity of the oil in stokes if the S.G of the oil 0.95

→ Solution:

Given- Each side of a square plate = 0.6 m

Area = 0.6 × 0.6 = 0.36 m^2

Thickness of oil film = dy = 12.5 mm = 12.5×10^{-3} m

Velocity of upper plate = u = 2.5 m/s

Force required on upper plate = F = 98.1 N

$$\therefore \text{Shear stress} = \tau = \frac{\text{Force}}{\text{Area}} = \frac{98.1}{0.36} \frac{N}{m^2}$$

$= 1592 \times \pi D \times L$

$= 1592 \times \pi \times 0.4 \times 90 \times 10^{-3}$

$F = 180.05\ N$

Torque on the shaft,

$T = \text{Force} \times \frac{D}{2}$

$= 180.05 \times \frac{0.4}{2}$

$T = 36.01\ Nm$

$\therefore$ Power lost $= \frac{2\pi N T}{60}$

$= \frac{2\pi \times 190 \times 36.01}{60}$

Power lost $= 716.48\ W$

---

20. If the velocity of profile of the fluid over a plate is parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/s, calculate the velocity gradient and shear stress at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 p...

→ Solution :

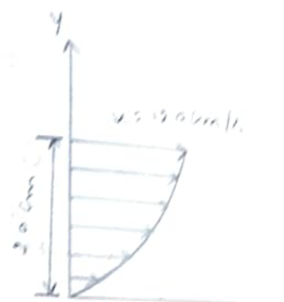
Given - Distance of vertex from plate $= 20\ cm$

velocity of vertex, $u = 120\ cm/s$

viscosity, $\mu = 8.5$ poise

$= \frac{8.5}{10} \frac{Ns}{m^2}$

$= 0.85\ Ns/m^2$



The velocity profile is given parabolic and equation of velocity profile is

$u = ay^2 + by + c$ —— ①

Where a, b and c are constants. Their values are determined from boundary conditions as:

(a) at $y = 0$, $u = 0$

(b) at $y = 20\text{ cm}$, $u = 120\text{ cm/sec}$

(c) at $y = 20\text{ cm}$, $\frac{du}{dy} = 0$

substituting boundary condition (a) in equation ① we get

$$\underline{C = 0}$$

Boundary condition (b) on substitution in ① , gives

$$120 = a(20)^2 + b(20)$$

$$120 = 400a + 20b \quad \text{—} \quad ②$$

Boundary condition (c) on substitution in ① gives

$$\frac{du}{dy} = 2ay + b$$

$$0 = 2 \times a \times 20 + b$$

$$0 = 40a + b \quad \text{—} \quad ③$$

Solving ② and ③

$$120 = 400a + 20b \quad \times 1$$

$$0 = 40a + b \quad \times 20$$

$$120 = 400a + \cancel{20b}$$

$$0 = 800a + \cancel{20b}$$

$$(-) \quad (-) \quad (-)$$

$$120 = -400a$$

$$\underline{a = -0.3}$$

$$120 = 400a + 20b$$

$$120 = 400(-0.3) + 20b$$

$$\underline{b = 12}$$

substituting the values of a, b and c in equation ①

$$\underline{u = -0.3y^2 + 12y}$$

velocity gradient , $\frac{du}{dy} = -0.3 \times 2y + 12$

$$\frac{du}{dy} = -0.6y + 12$$

at $y = 0$; $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 = 12/s$

at $y = 10$; $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = 6/s$

at $y = 20$; $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = 0$

Shear stress, $\tau = \mu\left(\frac{du}{dy}\right)$

(i) Shear stress at $y=0$, $\tau = \mu\left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.1$

(ii) Shear stress at $y=10$, $\tau = \mu\left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6 = 5.1\ N/m^2$

(iii) Shear stress at $y=20$, $\tau = \mu\left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 = 0$

Q1. Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surface at a speed of 0.6 m/s, if:
(i) thin plate is in the middle of the two plane surfaces
(ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces? Take the dynamic viscosity of glycerine $= 8.10 \times 10^{-1}\ Ns/m^2$

→ Solution :

Given - Distance between two large surfaces = 2.4 cm

Area of thin plate, $A = 0.5\ m^2$

velocity of thin plate, $u = 0.6\ m/s^2$

viscosity of glycerine $\mu = 8.10 \times 10^{-1}\ Ns/m^2$

Case I - When the thin plate is in the middle of the two plane surfaces

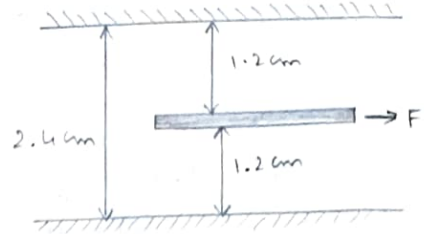
Let, F_1 = Shear force on the upper side of the thin plate

F_2 = Shear force on the lower side of the thin plate

F = Total force required to drag the plate

Then, $F = F_1 + F_2$

The shear stress (τ_1) on the upper side of the thin plate is given by,



$$\tau_1 = \mu\left(\frac{du}{dy}\right)_1$$

where, du = Relative velocity between thin plate and upper large plane surface = 0.6 m/s

dy = Distance between thin plate and upper large plane surface = 1.2 cm = 0.012 m
(plate is a thin one and hence thickness of plane is neglected)

$$\therefore \tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right)$$

$$\tau_1 = 40.5 \ N/m^2$$

Now, shear force F_1 = shear stress × Area

$= \tau_1 \times A$

$= 40.5 \times 0.5$

$\underline{F_1 = 20.25 N}$

similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = \mu\left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1}\left(\frac{0.6}{0.012}\right)$$

$$\tau_2 = 40.5 \ N/m^2$$

$\therefore$ Shear force, $F_2 = \tau_2 \times A$

$= 40.5 \times 0.5$

$\underline{F_2 = 20.25 N}$

$\therefore$ Total force = $F = F_1 + F_2$

$= 20.25 + 20.25$

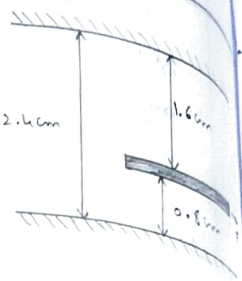
$\underline{F = 40.5 N}$

<u>Case II</u> - when the thin plate is at a distance of 0.8 cm from one of the plane surface.

Let, the thin plate is a distance of 0.8 cm from lower plane surface

$= 2.4 - 0.8 = 1.6\ cm = 0.16 m$

(neglecting thickness of the plate)



The shear force on the upper side of the thin plate,

$F_1 = \text{shear stress} \times \text{Area}$

$F_1 = \tau_1 \times A$

$= \mu \left(\frac{du}{dy}\right)_1 \times A$

$= 8.10 \times 10^{-1} \left(\frac{0.6}{0.016}\right) \times 0.5$

$\underline{F_1 = 15.18\ N}$

The shear force on the lower side of the thin plate

$F_2 = \tau_2 \times A$

$= \mu \left(\frac{du}{dy}\right)_2 \times A$

$= 8.10 \times 10^{-1} \left(\frac{0.6}{0.8/100}\right) \times 0.5$

$\underline{F_2 = 30.36\ N}$

$\therefore$ Total force required $= F = F_1 + F_2$

$= 15.18 + 30.36$

$\underline{F = 45.54\ N}$

22. Find the kinematic viscosity of an oil having density 981 kg/m³. The shear stress at a point in oil is 0.2452 N/m² and velocity gradient at that point is 0.2 per second

→ Solution:

Given - Mass density $= \rho = 981\ kg/m^3$

Shear stress $= \tau = 0.2452\ N/m^2$

velocity gradient $= \frac{du}{dy} = 0.2 /s$

$\tau = \mu \left(\frac{du}{dy}\right)$

$0.2452 = \mu \times 0.2$

$\mu = \frac{0.2452}{0.2}$

$\underline{\mu = 1.226\ Ns/m^2}$

Kinematic viscosity ν is

$\nu = \frac{\mu}{\rho} = \frac{1.226}{981}$

$= 0.125 \times 10^{-2}$

$= 0.125 \times 10^{-2}$

$= 0.125 \times 10^{2}$

$= 12.5\ cm^2/$

$\underline{\nu = 12.5\ stok}$

23 State and prove Pascals

→ It states that "th

at a point in a stat

directions".

Proof:

Consider an arbitra

fluid element of wedge

a fluid mass at rest

Let the width of the el

unity and P_x, P_y and P_z

pressure or intensity of P

on the face AB, AC an

Let $\angle ABC = \theta$. Then the fo

the element are:

1. Pressure forces
2. Weight of elem

$$\tau = \mu\left(\frac{du}{dy}\right)$$

$$0.2452 = \mu \times 0.2$$

$$\mu = \frac{0.2452}{0.2}$$

$$\mu = 1.226 \ Ns/m^2$$

Kinematic viscosity ν is given by

$$\nu = \frac{\mu}{\rho} = \frac{1.226}{981}$$

$$= 0.125 \times 10^{-2} \ m^2/s$$

$$= 0.125 \times 10^{-2} \times 10^{+4} \ cm^2/s$$

$$= 0.125 \times 10^{2} \ cm^2/s$$

$$= 12.5 \ cm^2/s \qquad [\because cm^2/s = \text{stoke}]$$

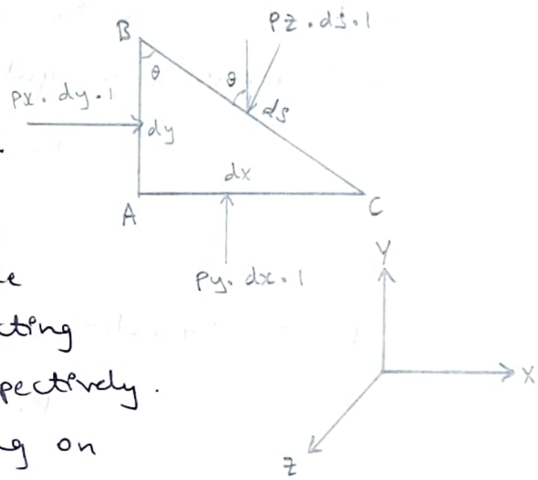
$$\nu = 12.5 \ \text{stokes}$$

Q3 State and Prove Pascal's Law

→ It states that "the pressure or intensity of pressure at a point in a static fluid is equal in all directions".

Proof:

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown.



Let the width of the element is unity and p_x, p_y and p_z are the pressure or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

1. Pressure forces normal to the surface.
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face AB = $p_x \times$ Area of face AB

$= p_x \times dy \times 1$

Similarly force on the face AC = $p_y \times dx \times 1$

Force on the face BC = $p_z \times ds \times 1$

weight of element = $\frac{AB \times AC}{2} \times 1 \times w$

where, w = weight density of fluid

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p_z (ds \times 1) \sin(90° - \theta) = 0$$

$$p_x \times dy \times 1 - p_z \times ds \times 1 \cos\theta = 0$$

but from figure, $ds \cos\theta = AB = dy$

$$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

$$p_x = p_z \quad — (1)$$

Similarly, resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos(90 - \theta) - \frac{dx \times dy}{2} \times 1 \times w = 0$$

$$p_y \times dx - p_z \, ds \sin\theta - \frac{dx \, dy}{2} \times w = 0$$

but $ds \sin\theta = dx$ and also the element is very small and hence weight is negligible

$$p_y \, dx - p_z \, dx = 0$$

$$p_y = p_z \quad — (2)$$

From equations (1) and (2), we have

$$\boxed{p_x = p_y = p_z}$$

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is same in all directions.

24. A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.

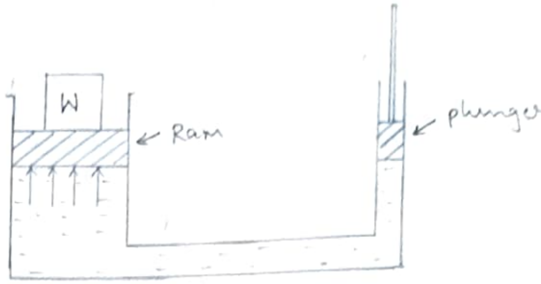
Solution:

Given - Diameter of ram,

$D = 30\text{ cm} = 0.3\text{ m}$

Diameter of plunger

$d = 4.5\text{ cm} = 0.045\text{ m}$



Force on plunger $F = 500\text{ N}$

Weight lifted $= W$

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068\text{ m}^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.045)^2 = 0.00159\text{ m}^2$

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{0.00159}\ \text{N/m}^2$$

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions, Hence the pressure intensity at the ram

$$= \frac{500}{0.00159} = 314465.4\ \text{N/m}^2$$

but Pressure intensity at ram $= \frac{\text{Weight}}{\text{Area of ram}} = \frac{W}{A}$

$$= \frac{W}{0.07068}\ \text{N/m}^2$$

$$\therefore \frac{W}{0.07068} = 314465.4$$

Lifted weight $= 314465.4 \times 0.07068$

$= 22222\text{ N}$

$\boxed{\text{weight} = 22.222\text{ kN}}$

25 An open tank contains water upto a depth of 2m and above it an oil of specific gravity 0.9 for a depth of. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.

→ Solution: given - Height of water, $Z_1 = 2m$

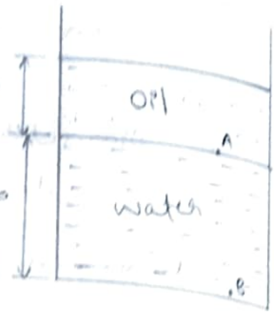
Height of oil, $Z_2 = 1m$

Specific gravity of oil, $S_0 = 0.9$

Density of water $= \rho_1 = 1000 kg/m^3$

Density of oil $= \rho_2 = 0.9 \times 1000$

$= 900 kg/m^3$



Pressure intensity at any point is given by

$$P = \rho \times g \times Z$$

(i) At interface i.e, at A

$$P = \rho_2 \times g \times 1.0$$

$$= 900 \times 9.8 \times 1.0$$

$$P = 8829 \ N/m^2$$

$$P = \frac{8829}{10^4} = 0.8829 \ N/cm^2$$

$$P = 0.8829 \ N/cm^2$$

(ii) At the bottom i.e, at B

$$P = \rho_2 \times g z_2 + \rho_1 \times g \times Z_1$$

$$= 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$

$$= 8829 + 19620$$

$$P = 28449 \ N/m^2$$

$$P = \frac{28449}{10^4} \ N/cm^2$$

$$\boxed{P = 2.8449 \ N/cm^2}$$

The diameters of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80N is applied on the small piston. Find the load lifted by the large piston when:

(a) the pistons are at the same level.

(b) small piston is 40 cm above the large piston

Solution: Given- Diameter of small piston, $d = 3\,cm$

$\therefore$ Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2$

$a = 7.068\ cm^2$

Diameter of large piston, $D = 10\,cm$

$\therefore$ Area of large piston, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (10)^2$

$A = 78.54\ cm^2$

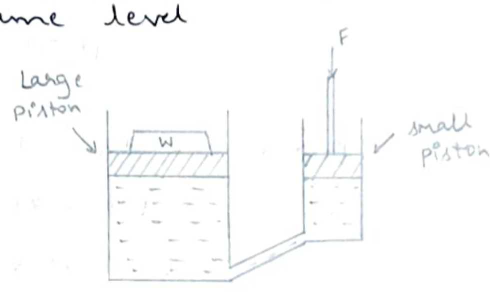
Force on small piston, $F = 80N$

Let the load lifted $= W$

(a) when the pistons are at same level

Pressure intensity on small piston

$\frac{F}{a} = \frac{80}{7.068}\ N/cm^2$



This is transmitted equally on the large piston

$\therefore$ Pressure intensity on the large piston $= \frac{80}{7.068}$

$\therefore$ Force on the large piston = Pressure X Area

$= \frac{80}{7.068} \times 78.54$

$= 888.96\ N$

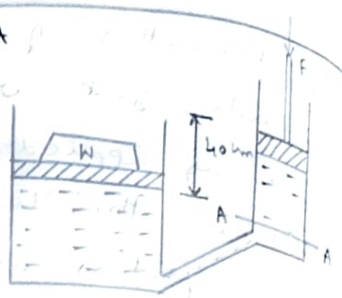
(b) when small piston is 40 cm above the large piston

Pressure intensity on the small piston

$= \frac{F}{a} = \frac{80}{7.068}\ N/cm^2$

$\therefore$ Pressure intensity at section A-A

$= \frac{F}{a}$ + pressure intensity due to height of 40cm of liquid



but, pressure intensity due to 40cm of liquid

$= \rho \times g \times h$

$= 1000 \times 9.81 \times 0.4 \ N/m^2$

$= \frac{1000 \times 9.81 \times 0.4}{10^4} \ N/cm^2$

$= 0.3924 \ N/cm^2$

$\therefore$ Pressure intensity at section A-A

$= \frac{80}{7.068} + 0.3924$

$= 11.32 + 0.3924$

$= 11.71 \ N/cm^2$

$\therefore$ Pressure intensity transmitted to the large piston

$= 11.71 \ N/cm^2$

$\therefore$ Force on the large piston

= pressure X Area of the large piston

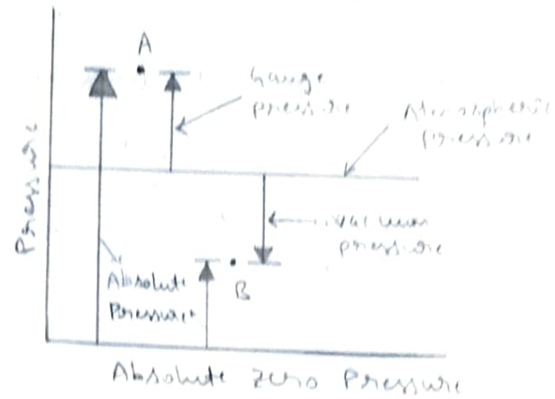
$= 11.71 \times A$

$= 11.71 \times 78.54$

$F = 919.7 \ N$

27. What is gauge pressure and vacuum pressure? Explain with formula

→ Gauge Pressure: It is defined as the pressure which is measured with the help of a pressure measuring instrum in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.



Vacuum Pressure : It is defined as the pressure below the atmospheric pressure. The relationship between the absolute pressure, gauge pressure and vacuum pressure

mathematically,

1) Absolute Pressure = Atmospheric pressure + Gauge pressure

$$\boxed{P_{abs} = P_{atm} + P_{gauge}}$$

2) vacuum pressure = Atmospheric pressure - Absolute pressure

---

28 A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m², calculate the new difference in the level of mercury. Sketch the arrangements in both cases.

→ Solution : $\underline{I^{st}}$ part

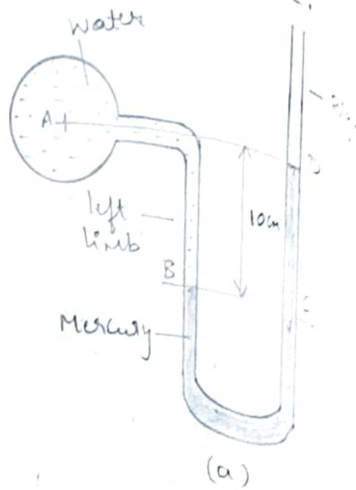
Given- Difference of mercury = 10 cm = 0.1 m

The arrangement is shown in figure (a)

Let, P_A = pressure of water in pipe line (i.e, at point A)

The points B and C lie on the same horizontal line. Hence pressure at B should be equal to pressure at C.

but pressure at B.



= pressure at A + pressure due to 10cm (0.1M) of water

$= P_A + \rho \times g \times h$

where, $\rho = 1000\ kg/m^3$ and $h = 0.1m$

$= P_A + 1000 \times 9.81 \times 0.1$

$= P_A + 981\ N/m^2$ — ①

Pressure at C = pressure at D + pressure due to 10cm of Mercury

$= 0 + \rho_0 \times g \times h_0$

where ρ_0 for mercury $= 13.6 \times 1000\ kg/m^3$ and $h_0 = 10cm = 0$

$\therefore$ Pressure at C $= 0 + (13.6 \times 1000) \times 9.81 \times 0.1$

$= 13341.6\ N$ — ②

But pressure at B is equal to pressure at C. Hence equating the equation ① and ②, we get

$P_A + 981 = 13341.6$

$P_A = 13341.6 - 981$

$\underline{P_A = 12360.6\ N/m^2}$

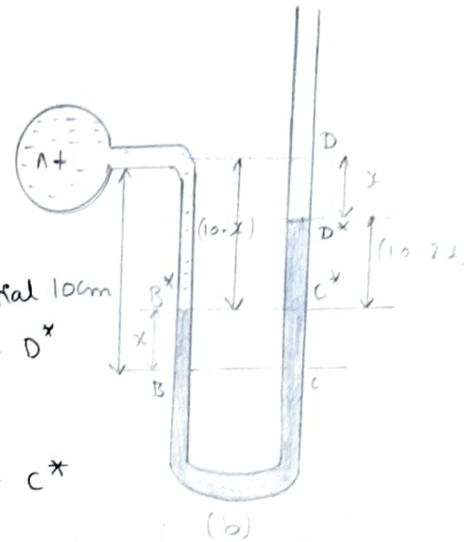
II nd Part

Given, $P_A = 9810\ N/m^2$

The arrangement is shown in figure (b). In this the pressure at A is $9810\ N/m^2$ which is less than $12360.6\ N/m^2$. Hence mercury in left limb will rise. rise of mercury in left limb will be equal to the fall mercury in right limb as the total volume of mercu remains same.

Let x = Rise of mercury in left limb in cm

Then fall of mercury in right limb $= x$ cm



The points B, C and D show the initial 10cm conditions whereas points B^*, C^* and D^* show the final condition.

The pressure at B^* = pressure at C^*

Pressure at A + pressure due to $(10-x)$ cm of water
= pressure at D^* + pressure due to $(10-2x)$ cm of mercury

$$P_A + \rho_1 \times g \times h_1 = P_{D^*} + \rho_2 \times g \times h_2$$

$$9810 + 1000 \times 9.81 \times \left(\frac{10-x}{100}\right) = 0 + (13.6 \times 1000) \times 9.81 \times \left(\frac{10-2x}{100}\right)$$

Dividing by 9.81, we get

$$1000 + 100 - 10x = 1360 - 272x$$

$$272x - 10x = 1360 - 1100$$

$$262x = 260$$

$$x = \frac{260}{262}$$

$$x = 0.992 \text{ cm}$$

$\therefore$ New difference of mercury $= (10 - 2x)$ cm

$= [10 - 2(0.992)]$

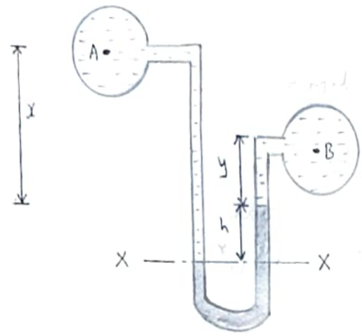
$= 8.016$ cm

---

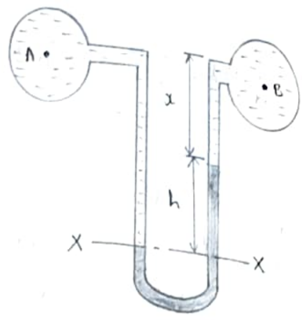
29. Derive the expression for U-tube differential manometer

→ Let the two points A and B are at different levels and also contains liquids of different specific gravity. These points are connected to the U-tube differential

Manometer. Let the pressure at A and B are P_A and P_B.



Two pipes at different levels



Two pipes at same level

(a)

Let, h = Difference of mercury level in the U-tube

y = Distance of the centre of B, from the mercury level in the right limb

x = Distance of the centre of A, from the mercury level in the right limb

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_g = Density of heavy liquid or mercury

Taking datum line at X-X

Pressure above X-X in the left limb = $\rho_1 g(h+x) + P_A$

where, P_A = Pressure at A

Pressure above X-X in the right limb = $\rho_g \times g \times h + \rho_2$

where P_B = Pressure at B

Equating the two pressures, we have

$$\rho_1 g(h+x) + P_A = \rho_g \times g \times h + \rho_2 g y + P_B$$

$$P_A - P_B = \rho_g gh + \rho_2 gh - \rho_1 g(h+x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

$\therefore$ Difference of pressure at A and B =

$$h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

(b) A and B are at the same level and contains the same liquid of density ρ_1. Then

Pressure above X-X in right limb $= \rho_g \times g \times h + \rho_1 g \times x + P_B$

Pressure above X-X in left limb $= \rho_1 \times g \times (h+x) + P_A$

Equating the two pressure

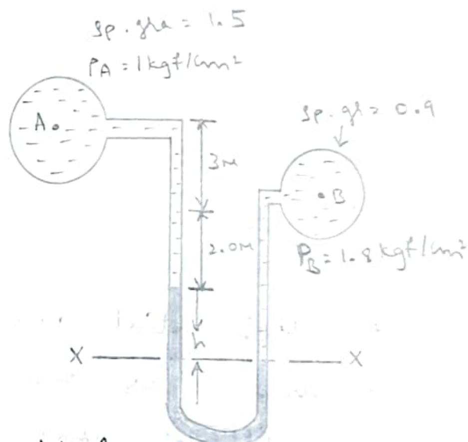
$$\rho_g \times g \times h + \rho_1 g x + P_B = \rho_1 \times g \times (h+x) + P_A$$

$$P_A - P_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g (h+x)$$

$$\underline{(P_A - P_B) = g \times h (\rho_g - \rho_1)}$$

30. A differential manometer is connected at the two points A and B of two pipes as shown in the figure. The pipe A contains a liquid of specific gravity 1.5, while pipe B contains a liquid of specific gravity 0.9. The pressures at A and B are 1 kgf/cm² and 1.80 kgf/cm² respectively. Find the difference in mercury level in the differential manometer

→ Solution: Given -



Specific gravity of liquid at A $S_1 = 1.5$

$\rho_1 = 1500$

Specific gravity of liquid at B $S_2 = 0.9$

$\rho_2 = 900$

Pressure at A, $P_A = 1 \text{ kgf/cm}^2$

$= 1 \times 10^4 \text{ kgf/m}^2$ $(\because 1 \text{ kgf} = 9.81 \text{ N})$

$= 10^4 \times 9.81 \text{ N/m}^2$

Pressure at B, $P_B = 1.8 \text{ kgf/cm}^2$

$= 1.8 \times 10^4 \text{ kgf/m}^2$

$= 1.8 \times 10^4 \times 9.81 \text{ N/m}^2$

Density of mercury $= 13.6 \times 1000 \text{ kg/m}^3$

Taking X–X as datum line

pressure above X–X in the left limb

$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2+3) + P_A$

$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$

Pressure above X – X in the right limb

$= 900 \times 9.81 \times (h+2) + P_B$

$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$

Equating the two pressures, we get

~~13.6h + 7.5~~

$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4 =$
$900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$

Dividing by 1000×9.81, we get

$13.6h + 7.5 + 10 = (h + 2.0) \times 0.9 + 18$

$13.6h + 17.5 = 0.9h + 1.8 + 18$

$13.6h + 17.5 = 0.9h + 19.8$

$(13.6 - 0.9)h = 19.8 - 17.5$

$12.7h = 2.3$

$h = \frac{2.3}{12.7}$

$h = 0.181 m$

$\boxed{h = 18.1 cm}$

31. A differential manometer is connected at the two points A and B as shown in the figure. At B air pressure is $9.81 N/cm^2$ (abs), find the absolute pressure at A

→ Solution: Given- Air pressure at $B = 9.81 N/cm^2$

$P_B = 9.81 \times 10^4 N/m^2$

Density of oil $= 0.9 \times 1000 = 900 kg/m^3$

Density of mercury $= 13.6 \times 1000 kg/m^3$

Let, The pressure at A be P_A

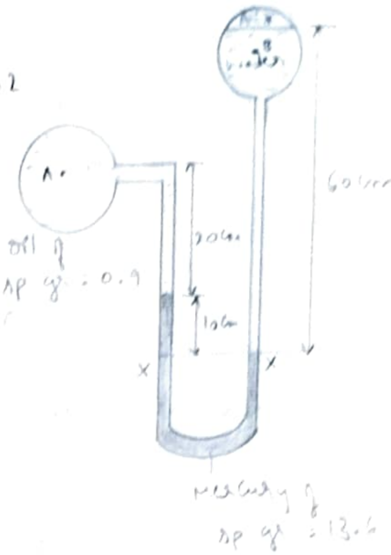
49

Taking datum line at X-X

Pressure above X-X in the left limb

$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + P_A$

$= 13341.6 + 1765.8 + P_A$



Pressure above X-X in the right limb

$= 1000 \times 9.81 \times 0.6 + P_B$

$= 5886 + 98100$

$= 103986$

Equating the two pressure head

$$103986 = 13341.6 + 1765.8 + P_A$$

$$P_A = 103986 - 15107.4$$

$$P_A = 88876.8$$

$$P_A = 88876.8 \ N/m^2$$

$$P_A = \frac{88876.8 \ N}{10000 \ cm^2}$$

$$\underline{P_A = 8.887 \ N/cm^2}$$

$\therefore$ Absolute Pressure at A $= \underline{8.887 \ N/cm^2}$

---

32 Water is flowing through different pipes to which an inverted differential manometer is connected having an oil of specific gravity 0.8, the pressure head in the pipe A is 2m of water, find P_B for the reading.

→ Solution: Given -

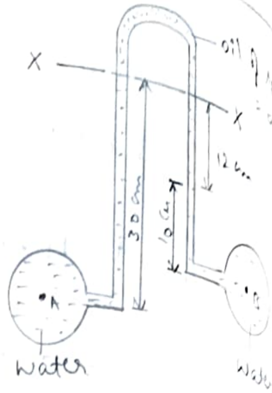
Pressure head at A $= \frac{P_A}{\rho g} = 2m$ of water

$\therefore \quad P_A = \rho \times g \times 2$

$= 1000 \times 9.81 \times 2$

$P_A = 19620 \ N/m^2$

Figure shows the arrangement



Taking X - X as datum line

Pressure below X - X in the left limb

$= P_A - \rho_1 \times g \times h_1$

$= 19620 - 1000 \times 9.81 \times 0.3$

$= 16677 \ N/m^2$

Pressure below X - X in the right limb

$= P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$

$= P_B - 981 - 941.76$

$= P_B - 1922.76$

Equating two pressures, we get

$16677 = P_B - 1922.76$

$P_B = 16677 + 1922.76$

$P_B = 18599.76 \ N/m^2$

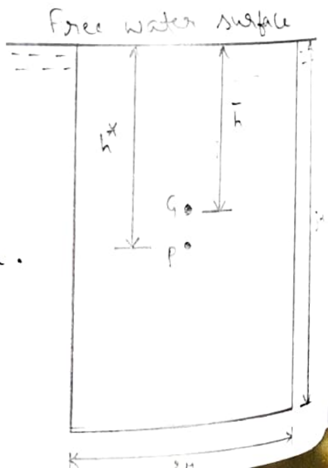
$\boxed{P_B = 1.8599 \ N/cm^2}$

33. A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge in horizontal and

(a) Coincides with water surface

(b) 2.5m below the free water surface

→ Solution: Given -



width of plane surface, $b = 2m$

Depth of plane surface, $d = 3m$

(a) upper edge coincides with water surface.

Total pressure is given by equation,

$F = \rho g A \bar{h}$

where, $\rho = 1000\ kg/m^3$ $\quad g = 9.81\ m/s^2$

$A = 3 \times 2 = 6\ m^2$, $\bar{h} = \frac{1}{2}(3) = 1.5\ m$

$F = 1000 \times 9.81 \times 6 \times 1.5$

$\underline{F = 88290\ N}$

Depth of Centre of pressure is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where, I_G = Moment of inertia about C.G of the area of surface

$$I_G = \frac{bd^3}{12} = \frac{2 \times (3)^3}{12} = 4.5\ m^4$$

$$\therefore h^* = \frac{4.5}{6 \times 1.5} + 1.5$$

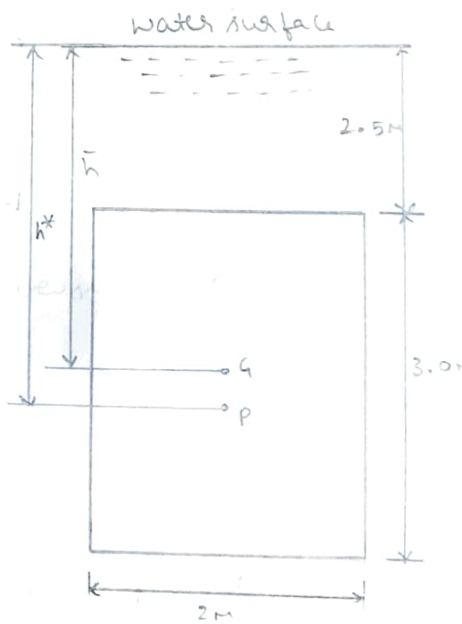
$$h^* = 0.5 + 1.5$$

$$\boxed{h^* = 2.0\ m}$$

(b) upper edge is 2.5 m below water surface

Total pressure (F) is given by

$$F = \rho g A \bar{h}$$



where $\bar{h}$ = Distance of C.G from free surface of water

$$\bar{h} = 2.5 + \frac{3}{2} = 4.0\ m$$

$F = 1000 \times 9.81 \times 6 \times 4.0$

$\underline{F = 235440\ N}$

Centre of pressure is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where, $I_G = 4.5$, $A = 6.0$, $\bar{h} = 4.0$

$$h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

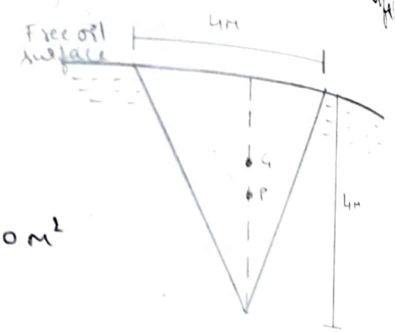
$$h^* = 0.1875 + 4.0$$

$$\boxed{h^* = 4.1875 \text{ m}}$$

34. Determine the total pressure and centre of pressure on an isosceles triangular plate of base 4m and altitude 4m when it is immersed vertically in an oil of specific gravity 0.9, the base of the plate coincides with the free surface of oil.

→ Solution: Given - Base of plate, $b = 4m$

Height of plate, $h = 4m$



$\therefore$ Area, $A = \frac{b \times h}{2} = \frac{4 \times 4}{2} = 8.0 m^2$

Specific gravity of oil, $S = 0.9$

$\therefore$ Density of oil, $\rho = 900 \text{ kg/m}^3$

The distance of C.G from the free surface of oil.

$$\bar{h} = \frac{1}{3} \times h$$

$$= \frac{1}{3} \times 4$$

$$\bar{h} = 1.33 m$$

Total pressure (F) is given by

$$F = \rho g A \bar{h}$$

$$= 900 \times 9.81 \times 8.0 \times 1.33$$

$$\underline{F = 9597.6 N}$$

Centre of pressure (h^*) from free surface of oil is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Where, I_G = Moment of inertia of triangular section about its C.G

$$I_G = \frac{bh^3}{36} = \frac{4\times(4)^3}{36}$$

$$I_G = 7.11\ m^4$$

$$\therefore h^* = \frac{7.11}{8.0\times 1.33} + 1.33$$

$$= 0.6667 + 1.33$$

$$\boxed{h^* = 1.99\ m}$$

35 A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper side of the aperture. The diagonals of the aperture are 2 m long and the tank contains a liquid of specific gravity 1.15. The centre of aperture is 1.5 m below the free surface. Calculate the thrust exerted on the plate by the liquid and position of its centre of pressure

→ Solution: Given - Diagonals of aperture, $AC = BD = 2m$

$\therefore$ Area of square aperture, A = area of ΔACB + Area of ΔACD

$$= \frac{AC\times BO}{2} + \frac{AC\times OD}{2}$$

$$= \frac{2\times 1}{2} + \frac{2\times 1}{2}$$

$$= 1+1$$

$$A = 2.0\ m^2$$

Specific gravity of liquid = 1.15

$\therefore$ Density of liquid = $\rho = 1.15\times 1000 = 1150\ kg/m^3$

Depth of centre of aperture from free surface,

$$\bar{h} = 1.5\ m$$

Where, I_G = Moment of inertia of triangular section about its C.G

$$I_G = \frac{bh^3}{36} = \frac{4\times(4)^3}{36}$$

$$I_G = 7.11\ m^4$$

$$\therefore h^* = \frac{7.11}{8.0 \times 1.33} + 1.33$$

$$= 0.6667 + 1.33$$

$$\boxed{h^* = 1.99\ m}$$

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$$= \frac{AC \times BO}{2} + \frac{AC \times OD}{2}$$

$$= \frac{2\times 1}{2} + \frac{2 \times 1}{2}$$

$$= 1 + 1$$

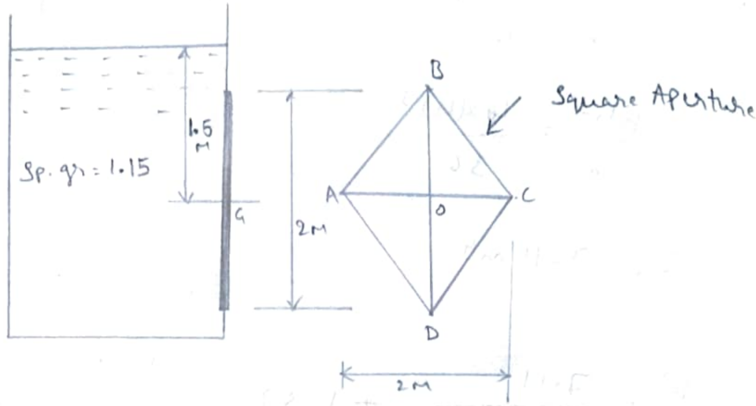
$$A = 2.0\ m^2$$

Specific gravity of liquid = 1.15

$\therefore$ Density of liquid = $\rho = 1.15 \times 1000 = 1150\ kg/m^3$

Depth of centre of aperture from free surface,

$$\bar{h} = 1.5\ m$$



(i) The thrust on the plate is given by

$$F = \rho A g \bar{h}$$

$$= 1150 \times 9.81 \times 2 \times 1.5$$

$$\boxed{F = 33844.5\ N}$$

(ii) Centre of pressure (h^*) is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = Moment of inertia of ABCD about diagonal AC

I_G = M.O.I of Δ ABO about AC + M.O.I of Δ ACD about

$$= \frac{AC \times OB^3}{12} + \frac{AC \times OD^3}{12}$$ [$\because$ M.O.I of a triangle about its base $= \frac{bh^3}{12}$

$$= \frac{2 \times (1)^3}{12} + \frac{2 \times (1)^3}{12}$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$I_G = \frac{1}{3}\ m^4$$

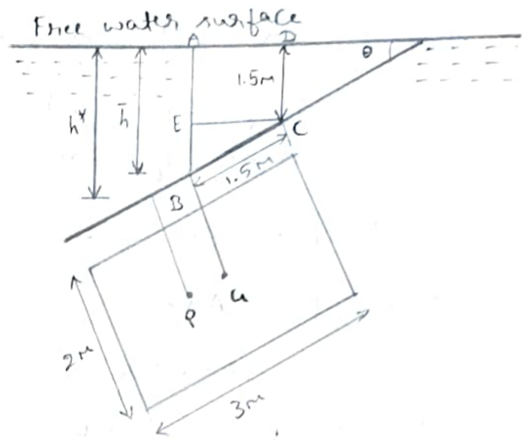
$$h^* = \frac{1/3}{2 \times 1.5} + 1.5$$

$$= \frac{1}{3 \times 2 \times 1.5} + 1.5$$

$$\boxed{h^* = 1.611\ m}$$

36 A rectangular plane surface 2m wide and 3m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge is 1.5m below the free water surface.

Solution : Given -



width of plane surface, b = 2m

Depth, d = 3m

Angle θ = 30°

Distance of upper edge from free water surface = 1.5m

(i) Total pressure force is given by

$$F = \rho g A \bar{h}$$

where, $\rho = 1000 \ kg/m^3$

$$A = b \times d = 3 \times 2 = 6m^2$$

$\therefore \bar{h}$ = Depth of C.G from free water surface

$$= 1.5 + 1.5 \sin 30°$$

$$\bar{h} = 2.25m$$

$\{\because \bar{h} = AE + EB = 1.5 + BC \sin 30$
$= 1.5 + 1.5 \sin 30\}$

$$\therefore F = 1000 \times 9.81 \times 6 \times 2.25$$

$$\boxed{F = 132435 \ N}$$

(ii) Centre of pressure (h^*)

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where, $I_G = \frac{bd^3}{12} = \frac{2 \times (3)^3}{12} = 4.5 \ m^4$

$$= \frac{4.5 \sin^2 30}{6 \times 2.25} + 2.25$$

$$= 0.0833 + 2.25$$

$$\boxed{h^* = 2.333 \ m}$$

37 A rectangular plane surface 3m wide and 4m deep lies water in such a way that its plane makes an angle 30° with the free surface. Determine the total pressure force and CP when the upper edge is 2m below the free surface

→ Solution : Given -

$b = 3m$, $d = 4m$, $\theta = 30°$

Distance of upper edge from free surface of water = 2m

Total pressure force is given by $F = \rho A g \bar{h}$

where, $\rho = 1000 \, kg/m^3$

$\bar{h} = 2 + BE = 2 + BC \sin\theta = 3m$

$A = b \times d = 12 m^2$

$\therefore$ The total pressure force

$F_1 = 1000 \times 9.81 \times 12 \times 3$

$F = 353167 N$

$\therefore F = 353.167 \, kN$

(ii) Centre of pressure (h^*)

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where, $I_G = \frac{bd^3}{12} = \frac{3 \times (4)^3}{12} = 16 m^4$

$$h^* = \frac{16 \sin^2 30}{12 \times 3} + 3$$

$$\boxed{h^* = 3.11m}$$

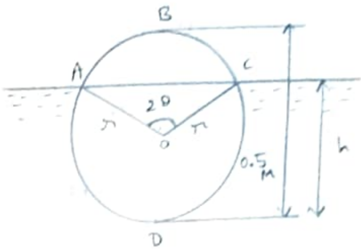
38. A wooden log of 0.6m diameter and 5m depth is floating river water. Find the depth of the wooden log in water when the specific gravity of the log is 0.7

→ Solution : Given - Diameter of log = 0.6m

Length, $L = 5m$

Specific gravity, $S = 0.7$

$\therefore$ Density of log $= 0.7 \times 1000$
$= 700 kg/m^3$



$\therefore$ weight density of log.
$w = \rho \times g$
$w = 700 \times 9.81$

To Find depth of immersion or h

weight of wooden leg = weight density $\times$ volume of log
$= 700 \times 9.81 \times \frac{\pi}{4}(D)^2 \times L$

$= 700 \times 9.81 \times \frac{\pi}{4}(0.6)^2 \times 5$

$= 989.6 \times 9.81$ N

For equilibrium,
weight of wooden log = weight of water displaced
= weight density of water $\times$ volume of water displaced

$\therefore$ volume of water displaced $= \frac{989.6 \times 9.81}{1000 \times 9.81} = 0.9896 \ m^3$

Let, h is the depth of immersion
$\therefore$ volume of log inside water = Area of ADCA $\times$ Length
= Area of ADCA $\times 5.0$

But volume of log inside water = volume of water displaced
$= 0.9896 \ m^3$

$0.9896 =$ Area of ADCA $\times 5.0$

$\therefore$ Area of ADCA $= \frac{0.9896}{5.0} = 0.1979 \ m^2$

But, Area of ADCA = Area of curved surface ADCOA + Area of Δ AOC
$= \pi r^2 \left[\frac{360 - 2\theta}{360}\right] + \frac{1}{2} r \cos\theta \cdot 2r\sin\theta$

$= \pi r^2 \left[1 - \frac{\theta}{180}\right] + r^2 \cos\theta \sin\theta$

$$0.1977 = \pi(0.3)^2\left[1 - \frac{\theta}{180}\right] + (0.3)^2 \cos\theta \sin\theta$$

$$0.1977 = 0.2827 - 0.00157\,\theta + 0.09\cos\theta\sin\theta$$

$$0.00157\,\theta - 0.09\cos\theta\sin\theta = 0.2827 - 0.1977$$

$$\theta - \frac{0.09}{0.00157}\cos\theta\sin\theta = \frac{0.0848}{0.00157}$$

$$\theta - 57.32\cos\theta\sin\theta = 54.01$$

$$\theta - 57.32\cos\theta\sin\theta - 54.01 = 0$$

For $\theta = 60°$, $60 - 57.32\cos 60 \sin 60 - 54.01 = -18.32$

For $\theta = 70°$, $70 - 57.32\cos 70 \sin 70 - 54.01 = -2.41$

For $\theta = 72°$; $72 - 57.32\cos 72 \sin 72 - 54.01 = 1.14$

For $\theta = 71°$; $71 - 57.32\cos 71 \sin 71 - 54.01 = -0.676$

$\therefore$ For $\theta = 71.5°$; $71.5 - 57.32\cos(71.5)\sin(71.5) - 54.01 = 0.43$

Then $h = r + r\cos\theta$

$= r + r\cos(71.5)$

$= 0.3 + 0.3\cos 71.5$

$$\boxed{h = 0.395\,m}$$

31 A body of dimensions 1.5m × 1.0m × 2m, weighs 1962 N in wa
Find its weight in air. what will be its specific gravi

→ Solution: Given:

volume of body $= 1.5 \times 1.0 \times 2.0 = 3.0\,m^3$

weight of body in water $= 1962$ N

volume of water displaced = volume of body = 3.0

$\therefore$ weight of water displaced $= 1000 \times 9.81 \times 3.0$

$= 29430$ N

For the equilibrium of the body

weight of body in air − weight of water displaced = weight in water

$$W_{air} - 29430 = 1962$$

$$W_{air} = 31392\,N$$

Mass of body $= \dfrac{\text{Weight in air}}{g} = \dfrac{31392}{9.81} = 3200\ kg$

Density of the body $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{3200}{3.0} = 1066.67\ kg/m^3$

$\therefore$ Specific gravity of the body $= \dfrac{1066.67}{1000} = \boxed{1.067}$

40 Find the density of a metallic body which floats at the interface of mercury of specific gravity 13.6 and water such that 40% of its volume is submerged in mercury and 60% in water

→ Solution: Given -

Let the volume of the body $= V\ m^3$

Then volume of body sub-merged in mercury $= \dfrac{40}{100} V = 0.4 V\ m^3$

volume of body sub-merged in water $= \dfrac{60}{100} V = 0.6 V m^3$

For the equilibrium of the body

Total buoyant force (upward force) = weight of the body

but, total buoyant force = Force of buoyancy due to water + Force of buoyancy due to mercury

Force of buoyancy due to water = weight of water displaced by body

= Density of water × g × volume of water displaced

= 1000 × g × volume of body in water

$= 1000 \times g \times 0.6\ V\ N$

Force of buoyancy due to mercury = weight of mercury displaced by body

= g × Density of mercury × volume of mercury displaced

= g × 13.6 × 1000 × volume of body in mercury

$= g \times 13.6 \times 1000 \times 0.4\ V\ N$

Weight of the body = Density × g × volume of body

$= \rho \times g \times V$

where, ρ = density of the body

∴ For equilibrium, we have

Total buoyant force = weight of the body

$1000 \times g \times 0.6V + 13.6 \times 1000 \times g \times 0.4V = \rho \times g \times V$

$\rho = 600 + 13600 \times 0.4.$

$= 600 + 54400$

$\rho = 6040.00 \; kg/m^3$

∴ Density of the body = $6040.00 \; kg/m^3$

END

ASSIGNMENT

1. Dimensional Analysis

Sl. No	Physical Quantity	Symbol	Dimension
	a) Fundamental		
1.	Length	L	L
2.	Mass	M	M
3.	Time	T	T
	b) Geometric		
4.	Area	A	L^2
5.	Volume	V	L^3
	c) Kinematic quantity		
6.	Velocity	v	LT^{-1}
7.	Angular velocity	ω	T^{-1}
8.	Acceleration	a	LT^{-2}
9.	Angular acceleration	α	T^{-2}
10.	Discharge	Q	L^3T^{-1}
11.	Acceleration due to gravity	g	LT^{-2}
12.	Kinematic viscosity	ν	L^2T^{-1}
	d) Dynamic quantity		
13.	Force	F	MLT^{-2}
14.	Weight	W	MLT^{-2}

Sl. No	Physical Quantity	Symbol	Dimension
15.	Density	ρ	ML^{-3}
16.	Specific weight	w	$ML^{-2}T^{-2}$
17.	Dynamic viscosity	μ	$ML^{-1}T^{-1}$
18.	Pressure intensity	P	$ML^{-1}T^{-2}$
19.	Modulus of elasticity	E	$ML^{-1}T^{-2}$
20.	Surface Tension	σ	MT^{-2}
21.	Shear stress		$ML^{-1}T^{-2}$
22.	work, Energy	w or E	$ML^{2}T^{-2}$
23.	Power	P	MLT^{-3}
24.	Torque	T	$ML^{2}T^{-2}$
25.	Momentum	M	MLT^{-1}

2. **Buckingham's Pi theorem**

The Rayleigh's method of dimensional analysis become more laborious if the variables are more than the number of fundamental dimensions (M, L, T). This difficulty is overcome by using Buckingham's π theorem which states " If there are n variables (independent and dependent) in a physical phenomenon and if variables contains m fundamental dimensions (M, L, T) then variables are arranged into (n-m) dimensionless terms. Each term is called π - term .

Let $x_1, x_2, x_3 \ldots x_n$ are the variables involved in a physical problem. Let x_1 be the dependent variable and $x_2, x_3 \ldots x_n$ are the independent variables on which it depends. Then x_1 is a function of $x_2, x_3 \ldots x_n$ and Mathematically expressed as

$$x_1 = f(x_2, x_3, \ldots x_n) \quad \text{— (1)}$$

$$f_1(x_1, x_2, x_3 \ldots x_n) = 0 \quad \text{— (2)}$$

Equation (1) is a dimensionally homogeneous equation. It contains n variables. If there are m fundamental dimensions then according to Buckingham's π theorem, equation (2) can be written in terms of number of dimensionless groups & π - terms in which number of π terms is equal to $(n-m)$.

Hence equation (2) becomes,

$$f(\pi_1, \pi_2, \ldots \pi_{n-m}) = 0 \quad \text{— (3)}$$

Each of π terms is dimensionless and independent of system. Division & multiplication by a constant does not change the constant character of π term. Each π term contains $m+1$ variables where m is the number of fundamental dimensions and is also called repeating variables.

Let in the above case x_2, x_3, x_4 are repeating variables if the fundamental dimension $m(M, L, T) = 3$

The each π - term is written as

$$\left.\begin{aligned}
\pi_1 &= x_2^{a_1} \cdot x_3^{b_1} \cdot x_4^{c_1} \cdot x_5^{d_1} \\
\pi_2 &= x_2^{a_2} \cdot x_3^{b_2} \cdot x_4^{c_2} \cdot x_5^{d_2} \\
\pi_{n-m} &= x_2^{a_{n-m}} \cdot x_3^{a_{n-m}} \cdot x_4^{a_{n-m}} \cdot x_5^{a_{n-m}}
\end{aligned}\right\} \text{— (4)}$$

Each equation is solved by the principle of dimensional homogeniety and values a, b, c are obtained. These values are substituted in equation (4) and values

$\pi_1, \pi_2 \ldots \pi_{n-m}$ are obtained. These values π are used in ③. The final equation for the phenomenon is obtained by expressing any one of π - terms as a functions of others as :

$$\left.\begin{aligned} \pi_1 &= \phi\,[\pi_2, \pi_3, \ldots \pi_{n-m}] \\ \pi_2 &= \phi_1\,[\pi_1, \pi_3 \ldots \pi_{n-m}] \end{aligned}\right\} - ⑤$$

3. **Dimensionless Number**

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force & gravity force & pressure force, surface tension & elastic force. As this is a ratio of one force to another, it will be dimensionless.

Important Dimensionless numbers

* Reynold's Number
* Froude's Number
* Euler's Number
* Weber's Number
* Mach's Number

1. **Reynold's Number** -

It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained as :

Inertia force (F_i) = Mass × Acceleration of flowing fluid

$$= \rho \times \text{volume} \times \frac{\text{velocity}}{\text{Time}}$$

$$= \rho \times \frac{\text{volume}}{\text{Time}} \times \text{velocity}$$

$$= \rho \times AV \times V$$

$\therefore$ volume per sec = Area × velocity = $A \times V$

$$F_i = \rho A V^2$$

Viscous force (F_v) = Shear stress × Area $\left\{ \because \tau = \mu \frac{du}{dy} \right.$

$= \tau \times A$ $\quad \therefore$ Force = τ × Area $\}$

$= \left(\mu \frac{du}{dy}\right) \times A$

$F_v = \mu \cdot \frac{V}{L} \times A$ $\quad \left\{ \because \frac{du}{dy} = \frac{V}{L} \right\}$

By definition, Reynolds number

$$Re = \frac{F_i}{F_v} = \frac{\rho A V^2}{\mu \cdot \frac{V}{L} \times A}$$

$$= \frac{\rho V L}{\mu}$$

$$= \frac{V \times L}{(\mu/\rho)}$$

$$\boxed{Re = \frac{V \times L}{\nu}}$$

$\left\{ \because \frac{\mu}{\rho} = \nu \right.$ = Kinematic viscosity$\}$

In case of pipe flow, the linear dimension L is taken as diameter, d. Hence Reynolds number for pipe flow,

$$Re = \frac{V \times d}{\nu} \quad \text{(or)} \quad Re = \frac{\rho V d}{\mu}$$

2. Froude's Number -

The Froudes Number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force. Mathematically, it is expressed as:

$$Fe = \sqrt{\frac{F_i}{F_g}}$$

WKT, $F_i = \rho A V^2$

and F_g = Force due to gravity

= Mass × Acceleration due to gravity

$= \rho \times \text{volume} \times g$

$= \rho \times L^2 \times g$ $\quad \{ \because \text{volume} = L^2 \}$

$$= \rho \times L^2 \times L \times g \qquad \{\because L^2 = A, \text{Area}\}$$

$$F_g = \rho \times A \times L \times g$$

$$F_e = \sqrt{\frac{F_i}{F_g}}$$

$$= \sqrt{\frac{\rho A V^2}{\rho A L g}}$$

$$= \sqrt{\frac{V^2}{L g}}$$

$$\boxed{F_e = \frac{V}{\sqrt{Lg}}}$$

3. <u>Euler's Number</u> -

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force. Mathematically, it is expressed as:

$$E_u = \sqrt{\frac{F_i}{F_p}}$$

where, F_p = Intensity of pressure × Area

$= P \times A$

and $F_i = \rho A V^2$

$$\therefore \quad E_u = \sqrt{\frac{\rho A V^2}{P A}}$$

$$= \sqrt{\frac{V^2}{P/\rho}}$$

$$\boxed{E_u = \frac{V}{\sqrt{P/\rho}}}$$

4. Weber's Number -

It is defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension force. Mathematically it is expressed as

$$\text{Weber's Number,} \quad W_e = \sqrt{\frac{F_i}{F_s}}$$

where, F_i = Inertia force = $\rho A V^2$

F_s = surface tension force

= surface tension per unit length × Length

$F_s = \sigma \times L$

$$W_e = \sqrt{\frac{\rho A V^2}{\sigma \times L}}$$

$$= \sqrt{\frac{\rho \times L^2 \times V^2}{\sigma \times L}}$$

$$= \sqrt{\frac{\rho L \times V^2}{\sigma}}$$

$$= \sqrt{\frac{V^2}{\sigma / \rho L}}$$

$$\boxed{W_e = \frac{V}{\sqrt{\sigma / \rho L}}}$$

5. Mach's Number -

Mach's number is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force. Mathematically, it is defined as:

$$M = \sqrt{\frac{\text{Inertia Force}}{\text{Elastic Force}}}$$

$$M = \sqrt{\frac{F_i}{F_e}}$$

where, $F_i = \rho A V^2$

F_e = Elastic force = Elastic stress X Area

$= K \times A$

$F_e = K \times L^2$

$$M = \sqrt{\frac{\rho A V^2}{K L^2}}$$

$$= \sqrt{\frac{\rho \times L^2 \times V^2}{K \times L^2}}$$

$$= \sqrt{\frac{V^2}{K/\rho}}$$

$$M = \frac{V}{\sqrt{K/\rho}}$$

But $\sqrt{\frac{K}{\rho}} = C$ = velocity of sound in the fluid

$$\therefore \boxed{M = \frac{V}{C}}$$

4. Eulers Equation of Motion :

This is equation of motion in which the f due to gravity and pressure are taken into considerat This is derived by considering the motion of a fluid element along a stream - line as:

Consider a stream - line in which flow is taking pla in s - direction as shown in the figure. consider a cylindrical element of cross sectional area dA and leng ds. The forces acting on the cylindrical element are

1. Pressure force $p\,dA$ in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direc of flow.
3. weight of element $\rho g\, dA\, ds$.

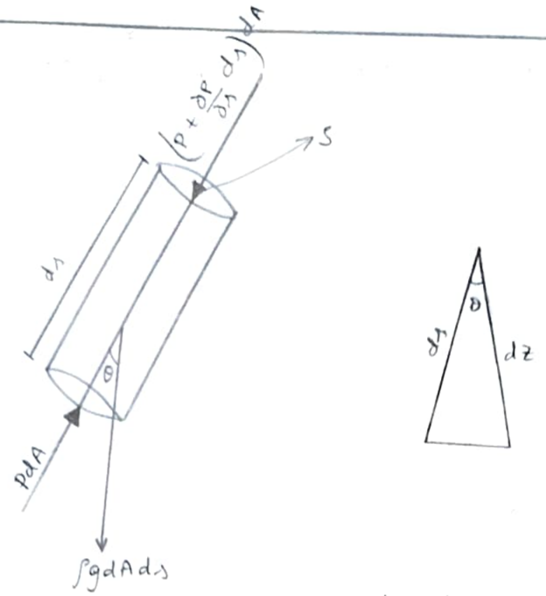


fig (a) fig (b)

Forces on a fluid element

Let θ be the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element × acceleration in the direction S.

$$\therefore \; pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta = \rho dA ds \times a_s \quad \text{①}$$

where, a_s = acceleration in the direction of S.

Now $a_s = \frac{dv}{dt}$, where v is a function of t and s

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}$$

$$a_s = \frac{v \partial v}{\partial s} + \frac{\partial v}{\partial t} \qquad \left\{\because \frac{ds}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$a_s = v \frac{\partial v}{\partial s}$$

Substituting the value of a_s in equation ① and simplifying the equation, we get

$$-\frac{\partial P}{\partial s}\, ds\, dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times \frac{v\,\partial v}{\partial s}$$

Dividing by $\rho\, ds\, dA$,

$$-\frac{\partial P}{\rho\,\partial s} - g\cos\theta = \frac{v\,\partial v}{\partial s}$$

$$\frac{\partial P}{\rho\,\partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

but, from figure (b), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \frac{1}{\rho}\frac{\partial P}{\partial s} + g\frac{dz}{ds} + v\frac{\partial v}{\partial s} = 0$$

$$\frac{\partial P}{\rho} + g\,dz\ v\,\partial v = 0 \quad \text{—} \quad (2)$$

$$\therefore \boxed{\frac{\partial P}{\rho} + g\,dz + v\,\partial v = 0}$$

The above equation is known as Euler's equation of motion.

Bernoulli's equation is obtained by integrating the Euler's equation of motion.

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \text{Constant}$$

If the flow is incompressible, ρ is constant and

$$\therefore \frac{P}{\rho} + gz + \frac{v^2}{2} = \text{Constant}$$

$$\frac{P}{\rho g} + z + \frac{v^2}{2g} = \text{Constant}$$

$$\boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{Constant}} \quad \text{—} \quad (3)$$

Equation (3) is a Bern

$\frac{P}{\rho g}$ = Pressure

Press

$\frac{v^2}{2g}$ = kinetic

z = Potential

Assumptions -

The following derivations of Berno

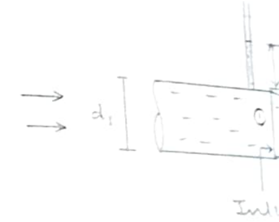
(i) The fluid is

(ii) The flow is

(iii) The flow is

(iv) The flow is

5. Expression for Rate



Consider a through which a in the figure.

Let d_1 = diame

P_1 = pressu

V_1 = veloc

a = area

and d_2, P_2, V_2

Applying Ber

$$-\frac{\partial P}{\partial s} ds dA - \rho g dA ds \cos\theta = \rho dA ds \times \frac{v \partial v}{\partial s}$$

Dividing by $\rho ds dA$,

$$-\frac{\partial P}{\rho \partial s} - g\cos\theta = \frac{v \partial v}{\partial s}$$

$$\frac{\partial P}{\rho \partial s} + g\cos\theta + \frac{v \partial v}{\partial s} = 0$$

but, from figure (b), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \frac{1}{\rho}\frac{\partial P}{\partial s} + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0$$

$$\frac{\partial P}{\rho} + g\,dz\; v\partial v = 0 \quad \text{—} \quad (2)$$

$$\therefore \boxed{\frac{\partial P}{\rho} + g\,dz + v\,\partial v = 0}$$

The above equation is known as Euler's equation of Motion.

Bernoulli's equation is obtained by integrating the Euler's equation of motion.

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \text{Constant}$$

If the flow is incompressible, ρ is constant and

$$\therefore \frac{P}{\rho} + gz + \frac{v^2}{2} = \text{Constant}$$

$$\frac{P}{\rho g} + z + \frac{v^2}{2g} = \text{Constant}$$

$$\boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{Constant}} \quad \text{—} \quad (3)$$

73

Equation ③ is a Bernoulli's equation in which

$\frac{P}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ = kinetic energy per unit weight or kinetic head

z = potential energy per unit weight or potential head

Assumptions -

The following are the assumptions made in the derivation of Bernoulli's equation

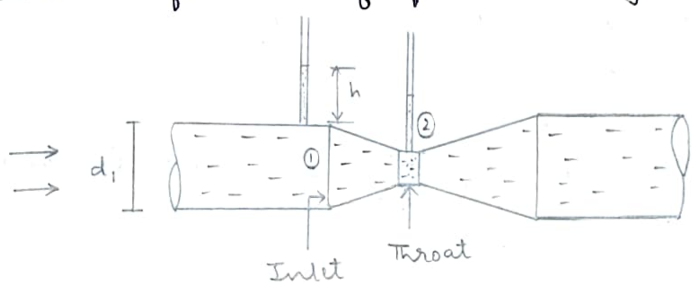
(i) The fluid is ideal i.e, viscosity is zero

(ii) The flow is steady

(iii) The flow is incompressible

(iv) The flow is irrotational

5. Expression for Rate of flow through venturimeter



Venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water) as shown in the figure.

Let d_1 = diameter at inlet or at section ①

P_1 = pressure at section ①

V_1 = velocity of fluid at section ①

a = area at section ① = $\frac{\pi}{4} d_1^2$

and d_2, P_2, V_2, a_2 are corresponding values at section ②

Applying Bernoulli's equation at sections ① and ②, we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As pipe is horizontal, hence $Z_1 = Z_2$

$$\therefore \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho g}$ is the difference of pressure heads at sectio ① and ② and it is equal to h & $\frac{P_1 - P_2}{\rho g} = h$

substituting this value of $\frac{P_1 - P_2}{\rho g}$ in the above equatio we get

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \text{— ①}$$

Now applying continuity equation at section ① and ②

$$a_1 V_1 = a_2 V_2$$

$$V_1 = \frac{a_2 V_2}{a_1}$$

substituting this value of V_1 in equation ①

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g}$$

$$= \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$= \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$V_2^2 = 2gh\left[\frac{a_1^2}{a_1^2 - a_2^2}\right]$$

$$V_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}}$$

$$V_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

∴ Discharge, $Q = a_2 V_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad — ②$$

Equation ② gives the discharge under ideal condition and is called theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\boxed{Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}}$$

where C_d = co-efficient of venturimeter and its value is less than 1.

6. **Orifice Meter :**

It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

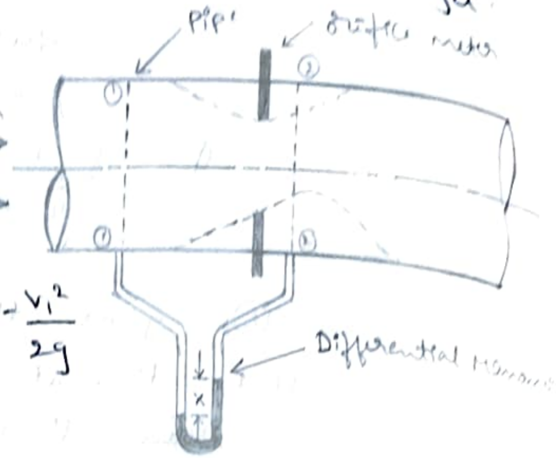
A differential manometer is connected at section ①, which is at distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section ②, which is at a distance of about half the diameter of the orifice on the down stream side from the orifice plate.

Let. p_1 = pressure at section ①

v_1 = velocity at section ①

a_1 = area of pipe at section ① and

p_2, v_2, a_2 are corresponding values at section ②. Applying Bernoulli's equation at sections ① and ②, we get



Orifice meter

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

but,

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = \text{Differential head}$$

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$2gh = v_2^2 - v_1^2$$

$$v_2 = \sqrt{v_1^2 + 2gh} \quad — ①$$

Now section ② is at the vena contract and a_2 represents the area at the vena contracta. If a_0 is the area of orifice then, we have

$$C_c = \frac{a_2}{a_0}$$

where, C_c = co-efficient of contraction

$$\therefore \quad a_2 = a_0 \times C_c \quad — ②$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2$$

$$v_1 = \frac{a_2 v_2}{a_1}$$

$$v_1 = \frac{a_o C_c v_2}{a_1} \quad — \quad (3)$$

substituting the value of v_1 in equation (1), we get

$$v_2 = \sqrt{2gh + \frac{a_o^2 C_c^2 v_2^2}{a_1^2}}$$

$$v_2^2 = 2gh + \left(\frac{a_o}{a_1}\right)^2 C_c^2 v_2^2$$

$$v_2^2 = \left[1 - \left(\frac{a_o}{a_1}\right)^2 C_c^2\right] + 2gh$$

$$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_o}{a_1}\right)^2 C_c^2}}$$

The discharge $Q = v_2 \times a_2$

$Q = v_2 \times a_o C_c$ [$\because$ from eq^n (2)]

$$Q = \frac{a_o C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_o}{a_1}\right)^2 C_c^2}} \quad — \quad (4)$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1 - \left(\frac{a_o}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_o}{a_1}\right)^2 C_c^2}}$$

$$C_c = C_d \frac{\sqrt{1 - \left(\frac{a_o}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_o}{a_1}\right)^2}}$$

substituting this value of C_c in equation (4), we get

$$Q = a_0 \times C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$Q = \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

$$\boxed{Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

where, C_d = coefficient of discharge for orifice meter

The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

7. Darcy - weisbach equation for loss of head due to friction in pipes

→ Consider a uniform horizontal pipe, having steady flow as shown in the figure. Let 1-1 and 2-2 are two sections of pipe.

Let, P_1 = pressure intensity at section 1-1

V_1 = velocity of flow at section 1-1

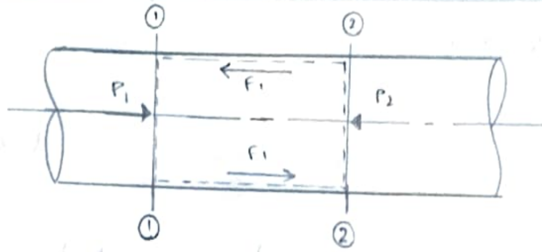
L = length of the pipe between sections 1-1 and 2-2.

d = diameter of pipe

f' = frictional resistance per unit wetted area per unit velocity

h_f = loss of head due to friction

and P_2, V_2 = are values of pressure intensity and velocity at section 2-2.



Uniform horizontal pipe

Applying Bernoulli's equations between section 1-1 and 2-2

Total head at 1-1 = Total head at 2-2 + loss of head due to friction b/w 1-1 and 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

but, $Z_1 = Z_2$ as pipe is horizontal

$V_1 = V_2$ as diameter of pipe is same at 1-1 and 2-2

$$\therefore \frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

$$h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \quad \text{— (1)}$$

But h_f is the head lost due to friction and hence the intensity of pressure will be reduced in the direction of flow by frictional resistance.

Now, Frictional Resistance = Frictional resistance per unit wetted area per unit velocity X wetted area X velocity

$F_1 = f' \times \pi d L \times V^2$ [$\because$ wetted area = $\pi d \times L$

$F_1 = f' \times P \times L \times V^2$ [$\because \pi d$ = Perimeter = P] velocity = $V = V_1 = V_2$]

$$\therefore F_1 = f' \times P \times L \times V^2 \quad \text{— (2)}$$

The forces acting on the fluid between sections 1-1 and 2-2 are:

1. Pressure force at section 1-1 and is equal to $P_1 \times A$

where, A = area of pipe

2. pressure force at section 2-2 $= P_2 \times A$

3. Frictional force F_1 as shown in the fig

Resolving all forces in the horizontal direction, we have,

$$P_1 A - P_2 A - F_1 = 0$$

$$(P_1 - P_2) A = F_1$$

$$(P_1 - P_2) A = f' \times P \times L \times V^2$$

$$(P_1 - P_2) = \frac{f' \times P \times L \times V^2}{A} \quad [\because \text{from } (2)]$$

but, from equation (1),

$$(P_1 - P_2) = \rho g h_f$$

equating the value of $(P_1 - P_2)$, we get

$$\rho g h_f = \frac{f' \times P \times L \times V^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad \text{— (3)}$$

In equation (3),

$$\frac{P}{A} = \frac{\text{Wetted Perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2$$

$$h_f = \frac{f'}{\rho g} \times \frac{4 L V^2}{d} \quad \text{— (4)}$$

Putting, $\frac{f'}{\rho g} = \frac{f}{2}$, where f is known as co-efficient of friction

Equation (4) becomes,

$$h_f = \frac{4.f}{2g} \times \frac{LV^2}{d}$$

$$\boxed{h_f = \frac{4fLV^2}{2g\times d}} \quad —— (5)$$

Equation (5) is known as Darcy - weisbach equation. The equation is commonly used for finding loss of head due to friction in pipes.

Sometimes equation (5) can also be written as

$$h_f = \frac{f \times L \times V^2}{2g \times d}$$

Then f is known as friction factor. or coefficient of friction coefficient of friction (f) which is a function of Reynold number

$$f = \frac{16}{Re} \quad \text{for } Re < 2000 \quad \text{(viscous flow)}$$

$$f = \frac{0.079}{Re^{1/4}} \quad \text{for Re varying from } 4000 \text{ to } 10^6$$

8. Chezy's formula for loss of head due to friction in pipes

→ WKT,
$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad —— (1).$$

where, h_f = loss of head due to friction

P = wetted perimeter of pipe

A = Area of cross-section of pipe

L = length of pipe

V = mean velocity of flow

Now, the ratio of $\frac{A}{P} = \left[\frac{\text{Area of flow}}{\text{Perimeter (wetted)}}\right]$ is called hydraulic depth or hydraulic radius and is denoted by m.

$\therefore$ Hydraulic mean depth,

$$m = \frac{A}{P} = \frac{\pi/4\, d^2}{\pi d} = \frac{d}{4}$$

substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation ①, we get

$$h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L \times V^2$$

$$V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L}$$

$$V^2 = \frac{h_f}{L} \times \frac{\rho g}{f'} \times m$$

$$\therefore \quad V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}}$$

$$V = \sqrt{\frac{\rho g}{f'}} \sqrt{m \frac{h_f}{L}} \quad — ②$$

Let, $\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as chezy's constant and $\frac{h_f}{L} = i$, where i is loss of head per unit length of pipe.

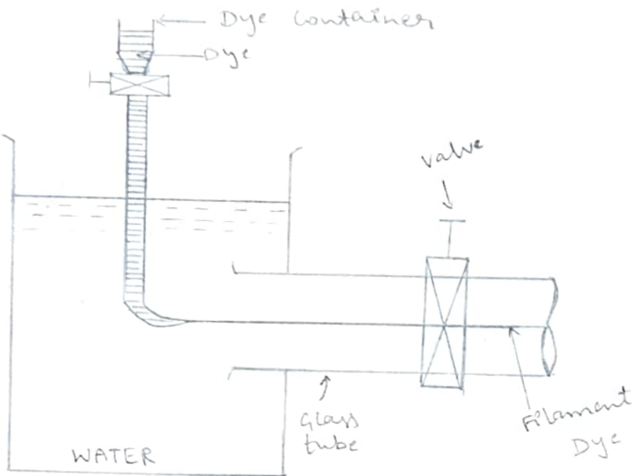
substituting the values $\sqrt{\frac{\rho g}{f'}}$ and $\sqrt{\frac{h_f}{L}}$ in equation ②, we get

$$\boxed{V = C\sqrt{mi}} \quad — ④$$

Equation ④, is known as chezy's formula. Thus the loss of head due to friction in pipe from chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. for a pipe the value of m is always equal to $d/4$.

9. **Reynold's Experiment :**

The type of flow is determined from the Reynold number i.e, $\frac{\rho v \times d}{\mu}$.



Reynold apparatus

The apparatus consists of :

(i) A tank containing water at constant head.

(ii) A small task containing some dye.

(iii) A glass tube having a bell-mouthed entrance at one end and a regulating valve at other ends

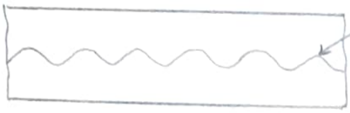
The water from the tank was allowed to flow through the glass tube. The velocity of flow was varied by the regulating valve. A liquid dye having same specific weight as water was introduced into the glass tube as shown in the figure above.

The following observations were made by Reynold:

(i) When the velocity of flow was low, the dye filament in the glass tube was in the form of a straight line. This straight line of dye filament was parallel to the glass tube, which was the case of laminar flow.

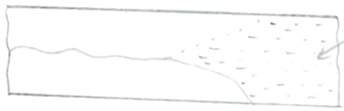
Laminar flow

(ii) with the increase of velocity of flow, the dye-filament was no longer a straight-line but it became a wavy one as shown in figure. This shows that flow is no longer laminar.



Transition

(iii) with the further increase of velocity of flow, the wavy dye-filament broke-up and finally diffused in water as shown in the figure. This means that fluid particles of the dye at this higher velocity are moving in random



Turbulent flow

fashion, which shows the case of turbulent flow. Thus in case of turbulent flow the mixing of dye-filament and water is intense and flow is irregular, random and disorderly.

In case of laminar flow, loss of pressure head was found to be proportional to the velocity but in case of turbulent flow, Reylond observed that loss of head is approximately proportional to the square of velocity. More exactly the loss of head, $h_f \propto V^n$, where n varies from 1.75 to 2.0

10. Experiments were conducted in a wind tunnel with a windspeed of 50 Km/hr on a flat plate of size 2m long and 1 m wide. The density of air is 1.15 kg/m^3. The coefficients of lift and drag are 0.75 and 0.15 respectively. Determine: ① The lift force ② Drag force ③ Resultant force ④ Direction of resultant force ⑤ Power exerted by air on the plate.

→ Solution: Given - Area of plate $A = 2m^2$

Velocity of air $= U = 50$ Km/hr $= 13.89$ m/s

Density of air = ρ = 1.15 kg/m^3
value of C_d = 0.15 , C_L = 0.75

① Lift force (F_L)

$$F_L = C_L \times A \times \rho \times \frac{V^2}{2}$$

$$= 0.75 \times 2 \times 1.15 \times \frac{(13.89)^2}{2}$$

$$\boxed{F_L = 166.604\ N}$$

② Drag force (F_D)

$$F_D = C_D \times A \times \rho \times \frac{V^2}{2}$$

$$= 0.15 \times 2 \times 1.15 \times \frac{(13.89)^2}{2}$$

$$\boxed{F_D = 33.28\ N}$$

③ Resultant force (F_R)

$$F_R = \sqrt{F_D^2 + F_L^2}$$

$$= \sqrt{(33.28)^2 + (166.604)^2}$$

$$\boxed{F_R = 169.67\ N}$$

④ Direction of resultant force (θ)

$$\tan\theta = \frac{F_L}{F_D}$$

$$= \frac{166.604}{33.28}$$

$$\boxed{\theta = 78.69^\circ}$$

⑤ Power exerted by air on the plate = Force in the direction of motion × velocity

$$= F_D \times v$$

$$= 33.28 \times 13.89$$

$$\boxed{P = 462.26\ W}$$

11. Find the difference in drag force exerted on a flat plate of size 2m X 2m when the plate is moving at a speed of $4 m s^{-1}$ normal to its plane in
① water ② air of density $1.24\ kg/m^3$, coefficient of drag = 1.15

→ Solution: Given - Area of plate = 2 X 2 = $4 m^2$

velocity of plate = $U = 4 m/s$

where $\rho = 1000 kg/m^3$ (water)

$C_D = 1.15$

① Drag force when in water

$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

$$= 1.15 \times 4 \times 1000 \times \frac{(4)^2}{2}$$

$$\boxed{F_D = 36800 N} \quad — ①$$

② Drag force when plate is moving in air of density = 1.24 kg/m³ →

$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

$$= 1.15 \times 4 \times 1.24 \times \frac{(4)^2}{2}$$

$$\boxed{F_D = 45.6 N} \quad — ②$$

Difference in drag force = 36800 − 45.6 = 36754.4 N

12. A truck having a projected area of $6.5 m^2$ travelling 70 km/hr has a total resistance of 2000N, of this 20% is due to rolling friction and 10% due to surface friction. The rest is from drag. Find the coefficient from drag. Take density of air $1.25 kg/m^3$

→ Solution: Given - Area of truck = $A = 6.5 m^2$

Speed of truck = U = 70 km/hr = 19.4 m/s

Total resistance = F_R = 2000N

Rolling friction = F_R = 20% = 400N

Surface friction = F_s = 10% = 200N

∴ From drag = 2000 − F_R − F_s = 2000 − 400 − 200 = 1400N

$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

where, F_D = form drag

C_D = coefficient of drag

$$\therefore F_D = C_D \times A \times \frac{\rho U^2}{2}$$

$$1400 = C_D \times 6.5 \times 1.25 \times \frac{(19.44)^2}{2}$$

$$\boxed{C_D = 0.912}$$

3. A circular disc 3m in diameter is held normal to a $26.4\ ms^{-1}$, wind of density 0.0012 gm/cc. what force is required to hold it at rest. Assume $C_D = 1.1$

Solution : Given - Diameter of disc = 3m

$U = 26.4\ ms^{-1}$

Area, $A = \frac{\pi (3)^2}{4} = 7.086\ m^2$

Density of wind $= 0.0012\ gm/cm^3 = 1.2\ kg/m^3$

$C_D = 1.1$

The force required to hold the body at rest = Drag exerted by wind on the disc

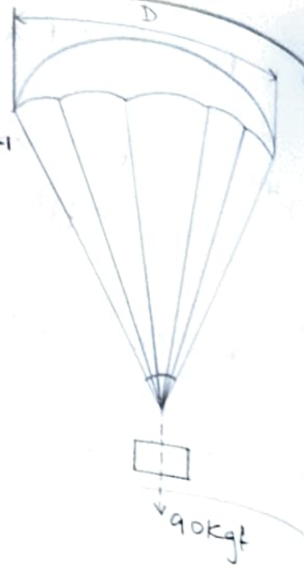
$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

$$= 1.1 \times 7.086 \times 1.2 \times \frac{(26.4)^2}{2}$$

$$\boxed{F_D = 3251.4\ N}$$

4. A man weighing 90 kgf descends to the ground from an aeroplane by a parachute against the air resistance. The velocity of coming down of the hemispherical parachute is 20 m/s. Find the diameter of parachute, $C_D = 0.5$ and $\rho_{air} = 1.25\ kg/m^3$

→ Solution: Given - weight of the man = 90 kgf = 882.9 N



weight of parachute = $u = 20\,ms^{-1}$

$C_D = 0.5$

$\rho_{air} = 1.25\ kg/m^3$

Let the diameter of parachute = D

$$Area = A = \frac{\pi D^2}{4}\ m^2$$

When the parachute with the man comes down with uniform velocity 20 m/s, the drag resistance will be equal to the weight of the man only. And projected area of the parachute will be $\frac{\pi D^2}{4}$

$\therefore$ Drag = F_D = 90 kgf = 90×9.81 = 882.9 N

using equation, $F_D = C_D \times A \times \frac{\rho U^2}{2}$

$$882.9 = 0.5 \times \frac{\pi D^2}{4} \times 1.25 \times \frac{(20)^2}{2}$$

$$D^2 = \sqrt{.9946}$$

$$\boxed{D = 2.999\ m}$$

15. A man weighing 981 N descends to the ground from a aeroplane with a parachute, hemispherical of 2m diameter. Find the velocity of that if its $C_D = 0.5$ and ρ_{air} = $0.00125\ g/cm^3$, $\nu = 0.015$ stoke

→ Solution: Given - $W = 981$ N

$\therefore$ Drag force = $F_D = W = 981$ N

Diameter of the parachute = $D = 2m$

Projected area = $A = \frac{\pi D^2}{4} = \frac{\pi (2)^2}{4} = \pi m^2$

$C_D = 0.5$

$\rho_{air} = 0.00125\ g/cm^3 = 1.25\ kg/m^3$

Let velocity = $U\ ms^{-1}$

$$F_D = C_D \times A \times \rho \times \frac{U^2}{2}$$

$$981 = 0.5 \times \pi \times 1.25 \times \frac{U^2}{2}$$

$$U = \sqrt{\frac{981 \times 2}{0.5 \times \pi \times 1.25}}$$

$$\boxed{U = 31.61\, ms^{-1}}$$

16. A man descends to the ground from aeroplane by a parachute of diameter 4m with a uniform velocity $U = 25 ms^{-1}$. Find the weight of the man if parachute is 9.81N. Take $C_D = 0.6$ and $\rho_{air} = 1.25\, Kg/m^3$

Solution: Given - Diameter = D = 4m

$$\text{Area} = A = \frac{\pi D^2}{4} = \frac{\pi (4)^2}{4} = 4\pi\, m^2$$

velocity of parachute $= U = 25 ms^{-1}$

weight of parachute $= w_1 = 9.81N$

$C_D = 0.6$

$\rho_{air} = 1.25\, Kg/m^3$

Let, the weight of man be W_2

weight of man $+ W_1 = W_2 + W_1 = W_2 + 9.81$ will be equal be drag force

$$\therefore F_D = C_D \times A \times \frac{\rho U^2}{2}$$

$$W_2 + 9.81 = 0.6 \times 4\pi \times 1.25 \times \frac{(25)^2}{2}$$

$$W_2 + 9.81 = 2945.24\, N$$

$$W_2 = 2945.24 - 9.81$$

$$\boxed{W_2 = 2935.43\, N}$$

17. Explain the basic equation for compressible flow

→ The basic equation of the compressible flow are

* Continuity equation
* Bernoulli's equation (or) energy equation
* Momentum equation
* Equation of station

1. Continuity equation :

This is based on the low of conservation of mass which states that matter cannot be created nor be destroyed but in other words, the matter of mass is constant. For one-d, steady flow, the mass per second = ρAV.

Where, ρ = mass density

A = Area of cross section

V = velocity

As mass or mass per second is constant. hence,

$$\rho AV = \text{constant} \quad -(1)$$

Differentiating

$$d(\rho AV) = 0, \quad \text{or} \quad \rho\, d(AV) + AV d\rho = 0$$

$$\rho\,[A dv + v dA] + AV d\rho = 0$$

$$\rho A dv + \rho v dA + AV d\rho = 0$$

÷ by ρAV,

$$\boxed{\frac{dv}{v} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0} \quad -(2)$$

Equation (2) is also called continuity equation in a differential form.

1. **Bernoulli's Equation:**

Bernoulli's equation has been derived for incompressible fluids. The same procedure is followed. The flow of fluid particles along a stream line in the direction of s is considered. The resultant force on the fluid particle in the direction of s is equated to the mass of the fluid particle and its acceleration. As the flow of compressible flow is steady, the same, Euler's equation is given by.

$$\frac{dP}{\rho} + v\,dv + g\,dz = 0 \quad \text{—} (1)$$

Integrating,

$$\int \frac{dP}{\rho} + \frac{v^2}{2} + gz = \text{constant}$$

In case of incompressible flow, the density ρ is constant and hence integration of $\frac{dP}{\rho}$ is equal to ρ. But in compressible flow, the ρ is unconstant. Hence ρ cannot be taken outside the integral sign. With the change of ρ, the pressure P also changes. The change of P and ρ take place according to equations depending on the type of process during flow. The value of ρ from these equations in terms of P is obtained and is substituted in $\int \frac{dP}{\rho}$ and then integrated. The Bernoulli's equation will be different for isothermal and adiabatic process.

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* **Bernoulli's equation for isothermal process**

For isothermal process, the relation between P and ρ is given by equation,

$$\frac{P}{\rho} = \text{constant} = C_1$$

$\therefore \quad \rho = \frac{P}{C_1}$

Hence, $\int \frac{dP}{\rho} = \int \frac{dP}{P/C_1} = \int \frac{C_1 dP}{P} = C_1 \int \frac{dP}{P}$

$= C_1 \log_e P = \frac{P}{\rho} \log_e P \qquad \left[C_1 = \frac{P}{\rho}\right]$

using in Bernoulli's equation

$$\frac{P}{\rho} \log_e P + \frac{v^2}{2} + gz = \text{constant}$$

$\div$ by g

$$\boxed{\frac{P}{\rho g} \log_e P + \frac{v^2}{2g} + z = \text{constant}}$$

This is the equation for compressible flow of isothermal process

$$\frac{P_1}{\rho_1 g} \log_e P_1 + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho_2 g} \log_e P_2 + \frac{v_2^2}{2g} + z_2$$

---

* **Bernoulli's equation for adiabatic process**

For adiabatic process, the relation between P & ρ is given by

$$\frac{P}{\rho^k} = \text{constant} = C_2$$

$$\rho^k = \frac{P}{C_2} \quad (\text{or}) \quad \rho = \left(\frac{P}{C_2}\right)^{1/k}$$

Hence, $\int \frac{dP}{\rho} = \int \frac{dP}{(P/C_2)^{1/k}} = \int \frac{(C_2)^{1/k}}{P^{1/k}} dP$

$= C_2^{1/k} \int \frac{1}{P^{1/k}} dP = C_2^{1/k} \int P^{-1/k} dP$

$= C_2^{1/k} \frac{P^{(-1/k+1)}}{-1/k+1} = \frac{C_2^{1/k} P^{\left(\frac{k-1}{k}\right)}}{\left(\frac{k-1}{k}\right)}$

$$= \frac{k}{(k-1)} \; C_2^{1/k} \; p^{\left(\frac{k-1}{k}\right)}$$

$$= \frac{k}{(k-1)} \left[\frac{p}{(\rho^k)}\right]^{1/k} p^{\left(\frac{1-k}{k}\right)} \qquad \left[C_2^{1/k} = \frac{p}{\rho}\right]$$

$$= \frac{k}{(k-1)} \; \frac{p^{1/k}}{\rho} \; p^{\left(\frac{1-k}{k}\right)} = \frac{k}{k-1} \; \frac{p}{\rho}$$

using this value,

$$\frac{k}{(k-1)} \frac{p}{\rho} + \frac{V^2}{2} + gz = \text{Constant}$$

$\div$ by g

$$\boxed{\frac{k}{(k-1)} \frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{Constant}} \quad \text{— (2)}$$

This is the Bernoulli's equation for compressible flow of adiabatic process.

$$\frac{k}{(k-1)} \frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{k}{(k-1)} \frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2$$

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8. Calculate the Mach number at a point on a jet propelled aircraft, which is at 1100 Km/hr at sea level where T = 20°C, Take k = 1.4 and R = 287.5 J/kgK.

→ Solution: Given- Speed of aircraft = V = 1100 Km/hr = 305.55 m/s

$t = 20°C$

$T = 273 + 20 = 293 K$

$k = 1.4$

$R = 287.5 \; J/kgK$

$$C = \sqrt{kRT}$$

$$= \sqrt{1.4 \times 287.5 \times 293}$$

$$C = 343.11 \; m/s$$

Mach number is given by

$$M = \frac{V}{c}$$

$$M = \frac{305.55}{343.11}$$

$$\boxed{M = 0.89}$$

19. An aeroplane is flying at 15 Km where $t = -50°c$. The speed is to $M = 2$. Assuming $K = 1.4$ and $R = 287.5$ J/kgK. Find the speed of the plane

→ Solution: Given- Height of the plane = $Z = 15$ Km
Temperature $t = -50°c$
$T = -50 + 273 = 223k$

Mach number; $M = 2$; $K = 1.4$; $R = 287.5$ J/kgk

WKT, $c = \sqrt{KRT}$

$= \sqrt{1.4 \times 287.5 \times 223}$

$$\boxed{c = 299.33 \text{ m/s}}$$

$$M = \frac{V}{c}$$

$$2 = \frac{V}{299.33}$$

$V = 2155.17$ Km/hr

$$\boxed{V = 598.66 \text{ m/s}}$$

20. A projectile is travelling in air having pressure and temperature as 8.829 N/cm^2 and $-2°c$. If the mach angle is $40°$. Find V. Take $K = 1.4$ and $R = 287.5$ J/kgK

→ Solution: Given- Pressure of air = $P = 8.829$ N/cm^2
$= 8.829 \times 10^4$ N/m^2

Temperature of air = $t = -2°c$
$T = -2 + 273 = 271K$

Mach number is given by

$$M = \frac{V}{c}$$

$$M = \frac{305.55}{343.11}$$

$$\boxed{M = 0.89}$$

19. An aeroplane is flying at 15Km where $t = -50°c$ The speed is to $M = 2$. Assuming $K = 1.4$ and $R = 287.5$ J
Find the speed of the plane

→ Solution: Given- Height of the plane = $Z = 15$Km
Temperature $t = -50°c$
$T = -50 + 273 = 223K$

Mach number; $M = 2$; $K = 1.4$; $R = 287.5$ J/kgk

WKT,

$$C = \sqrt{KRT}$$

$$= \sqrt{1.4 \times 287.5 \times 223}$$

$$\boxed{C = 299.33 \text{ m/s}}$$

$$M = \frac{V}{c}$$

$$2 = \frac{V}{299.33}$$

$V = 2155.17$ Km/hr

$$\boxed{V = 598.66 \text{ m/s}}$$

20. A projectile is travelling in air having pressure and temperature as 8.829 N/cm² and $-2°c$. If the mach angle is $40°$. Find V. Take $K = 1.4$ and $R = 287.5$ J/kg

→ Solution: Given- Pressure of air = $P = 8.829$ N/cm²
$= 8.829 \times 10^4$ N/m²

Temperature of air = $t = -2°c$
$T = -2 + 273 = 271K$

Mach angle $= \alpha = 40°$; $k = 1.4$; $R = 287.5$ J/kgk

WKT, $\sin \alpha = \frac{c}{v}$

$\sin 40 = \frac{c}{v}$

The velocity of s , c is given by

$c = \sqrt{kRT}$

$= \sqrt{1.4 \times 287.5 \times 271}$

$c = 329.8 \simeq 330$ m/s

$\therefore c = 330$ m/s

$\sin 40 = \frac{330}{v}$

$v = \frac{330}{\sin 40}$

$\boxed{v = 513 \text{ m/s}}$

21. A projectile travels in air of pressure 10.1043 N/cm³ at 10°c at a speed of 1500 km/hr. Find the Mach number and mach angle. Take $k = 1.4$ and $R = 287$ J/kgk

→ Solution : Given - Pressure $= P = 10.1043$ N/cm³

$P = 10.1043 \times 10^4$ N/m³

Temperature $= t = 10°c$

$T = 10 + 273 = 283$ K

Speed of projectile $= V = 1500$ km/hr

$V = 416.67$ ms⁻¹

$k = 1.4$; $R = 287$ J/kgk

For adiabatic process, the velocity of ...

$c = \sqrt{kRT}$

$= \sqrt{1.4 \times 287 \times 283}$

$c = 337.2$ m/s

Mach number $= M = \frac{v}{c}$

$$= \frac{416.67}{337.2}$$

$$\boxed{M = 1.235}$$

Mach angle $= \sin\alpha = \frac{c}{V}$

$$\sin\alpha = \frac{1}{M}$$

$$= \frac{1}{1.235}$$

$$\sin\alpha = 0.8097$$

$$\boxed{\alpha = 54.06^\circ}$$

22. A pipe through which water flows is having two diameters 20 cm and 10 cm at cross sections 1 and 2 respectively. The velocity of water at 1 is 4 m/s. Find velocity head at section 1 and 2 and also rate of discharge.

→ Solution: Given- $D_1 = 20\,cm = 0.2\,m$

$$A_1 = \frac{\pi}{4}(0.2)^2 = 0.0314\,m^2$$

$$V_1 = 4\,m/s$$

$$D_2 = 10\,cm = 0.1\,m$$

$$A_1 = \frac{\pi}{4}(0.1)^2 = 0.00785\,m^2$$

$$V_2 = ?$$

① velocity head at 1

$$= \frac{V_1^2}{2g} = \frac{(4)^2}{2\times 9.81} = 0.815\,m$$

② velocity head at 2 $= \frac{V_2^2}{2g}$

Apply equation of continuity at ① and ②

$$\therefore A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2}$$

$$V_2 = \frac{0.00314 \times 4}{0.00785}$$

$$V_2 = 16\, m/s$$

$\therefore$ velocity head at 2 $= \frac{V_2^2}{2g} = \frac{(16)^2}{2 \times 9.81} = \underline{83.047\, m}$

Rate of discharge

$$Q = A_1 V_1$$

$$= 0.0314 \times 4$$

$$Q = 0.1256\, m^3/s$$

$$\boxed{Q = 125.6 \text{ litres/s}}$$

END