Operations Research & Statistics Lab IP610L

Topics covered in Even Semester 2019-20 for 6th Semester IP students

1. Formulation of Linear Programming Problem
2. Graphical Method for solving LPP
3. Simplex Method for solving LPP
4. Big-M method for solving LPP
5. Converting Primal problem to Dual problem
6. Dual Simplex method for solving LPP
7. NWCM in Transportation problem
8. LCM in Transportation problem
9. VAM in Transportation problem
10. Regression Analysis

Note: Write the lab report for the problems solved in the lab during the even semester in white paper and submit the scanned copy by email to dayakar.devaru@jssstuniv.in before July 31st 2020.

The problems solved in the lab are given in this document for reference. Each problem carries 5 Marks.

1. Formulation of Linear Programming Problem

Note: We cannot fulfill all the demand forecast with the available inventory. The objective of this problem is to maximize the sales with the given inventory and trying to meet the demand forecast.

- Given are the final assembly (A1, A2, A3, A4, A5) that are to be sold.
- listed are the corresponding components that make the assembly

		Assembly				
		A1	A2	A3	A4	A5
No of Components	S1	1	1		2	
	S2	2			1	
	S3	1		1	3	1
	S4	2	1	1	1	1
	S5	1	2	1		

- List of the inventory and the selling cost for each components.
- * Assume assembly selling c ost is equal to the sum of all the components' selling cost.

Components	Stock	Cost ($)
S1	30	35
S2	5	24
S3	16	93
S4	20	30
S5	3	16

- Demand forecast for each assembly:

Assembly	Demand
A1	2
A2	4
A3	4
A4	5
A5	8

Questions to be answered:

Maximum sales value $:

____

ASSEMBLY.

	A_1	A_2	A_3	A_4	A_5	Stock	Cost($)
S_1	1	1		2		30	35
S_2	2			1		5	24
S_3	1		1	3	1	16	93
S_4	2	1	1	1	1	20	30
S_5	1	2	1			3	16

Assembly	Demand	Total Cost ($)
A_1	2	252
A_2	4	97
A_3	4	139
A_4	5	403
A_5	8	123

Let x_1, x_2, x_3, x_4 & x_5 be the number of assemblies of A_1, A_2, A_3, A_4 & A_5 to be manufactured.

obj fn $\Rightarrow$ max $\Rightarrow 252x_1 + 97x_2 + 139x_3 + 403x_4 + 123x_5$

S.t. Constraints: $x_1 + x_2 + 2x_4 \leq 30$

$$2x_1 + x_4 \leq 5$$

$$x_1 + x_3 + 3x_4 + x_5 \leq 16$$

$$2x_1 + x_2 + x_3 + x_4 + x_5 \leq 20$$

$$x_1 + 2x_2 + x_3 \leq 3$$

$x_1 \leq 2,\ x_2 \leq 4,\ x_3 \leq 4,\ x_4 \leq 5,\ x_5 \leq 8$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

	x1	x2	x3	x4	x5		
	1	1	0	3	6	Target	
	252	97	139	403	123	**2296**	
S1 Constraint	1	1	0	2	0	8	30
S2 Constraint	2	0	0	1	0	5	5
S3Constraint	1	0	1	3	1	16	16
S4Constraint	2	1	1	1	1	12	20
S5 Constraint	1	2	1	0	0	3	3
A1 Constriant	1	0	0	0	0	1	2
A2 Constriant	0	1	0	0	0	1	4
A3 Constriant	0	0	1	0	0	0	4
A4 Constriant	0	0	0	1	0	3	5
A5 Constriant	0	0	0	0	1	6	8

2. Graphical Method for solving LPP

Old machines can be brought at Rs. 2 lakhs each & new machines at Rs. 5 lakhs each. The old machines produce 3 components/week while the new machines produce 5 components/week, each component being worth Rs, 30,000. A machine (New or old) costs Rs. 1 lakh/week to maintain. The company has only Rs. 80 lakhs to spend on machines. How many of each kind should the company buy to get a profit of more than Rs. 6 lakhs/week. Assume that the company cannot house more than 20 machines. Formulate this as a linear programming problem and solve it by graphical method.

3. Use Simplex Method for solving LPP

$$Max\,Z = 40x_1 + 50x_2$$
$$stc \quad 2x_1 + 3x_2 \le 3$$
$$8x_1 + 4x_2 \le 5$$
$$x_1, x_2 \ge 0$$

4. Use Big-M method for solving LPP

$$Max\,Z = 2x_1 + x_2$$
$$stc \quad 3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \le 4$$
$$x_1, x_2 \ge 0$$

5. Obtain the Dual problem of the following Primal problem

$$Minimize\,Z = 2x_1 - 5x_2 - 2x_3$$
$$stc \quad 3x_1 - x_2 + 2x_3 \le 9$$
$$2x_1 - 4x_2 \ge 14$$
$$-4x_1 + 3x_2 + 8x_3 = 12$$
$$x_1, x_2 \ge 0 \; and \; x_3 \; is \, unrestricted$$

6. Use Dual Simplex method for solving LPP

Max Z = $-2X_1 - X_2$

Stc. $3X_1 + X_2 \ge 3$

$4X_1 + 3X_2 \ge 6$

$X_1 + 2X_2 \ge 3$

$X_1, X_2 \ge 0$

7. Solve the following transportation problem by using VAM and optimize using UV Method.

	D1	D2	D3	D4	Supply
S1	42	48	38	37	16
S2	40	49	52	51	15
S3	39	38	40	43	19
Demand	8	9	11	16	

8. Solve the following transportation problem by using LCM and optimize using UV Method.

	D1	D2	D3	D4	Supply
S1	10	2	20	11	15
S2	12	7	9	20	25
S3	4	14	16	18	10
Demand	5	15	15	15	

9. Solve the following transportation problem by using NWCM and optimize using UV Method.

	D1	D2	D3	D4	Supply
S1	3	1	7	4	250
S2	2	6	5	9	350
S3	8	3	3	2	400
Demand	200	300	350	150	

10. Regression Analysis

How to Find the Regression Equation

In the table below, the x_i column shows scores on the aptitude test. Similarly, the y_i column shows statistics grades. The last two columns show deviations scores - the difference between the student's score and the average score on each test. The last two rows show sums and mean scores that we will use to conduct the regression analysis.

Student	x_i	y_i	$(x_i-\bar{x})$	$(y_i-\bar{y})$
1	95	85	17	8
2	85	95	7	18
3	80	70	2	-7
4	70	65	-8	-12
5	60	70	-18	-7
Sum	390	385		
Mean	78	77		

And for each student, we also need to compute the squares of the deviation scores (the last two columns in the table below).

Student	x_i	y_i	$(x_i-\bar{x})^2$	$(y_i-\bar{y})^2$
1	95	85	289	64
2	85	95	49	324
3	80	70	4	49
4	70	65	64	144
5	60	70	324	49
Sum	390	385	730	630
Mean	78	77		

And finally, for each student, we need to compute the product of the deviation scores.

Student	x_i	y_i	$(x_i-\bar{x})(y_i-\bar{y})$
1	95	85	136
2	85	95	126
3	80	70	-14
4	70	65	96
5	60	70	126
Sum	390	385	470
Mean	78	77	

The regression equation is a linear equation of the form: $\hat{y} = b_0 + b_1x$. To conduct a regression analysis, we need to solve for b_0 and b_1. Computations are shown below. Notice that all of our inputs for the regression analysis come from the above three tables.

First, we solve for the regression coefficient (b_1):

$$b_1 = \Sigma [(x_i - \bar{x})(y_i - \bar{y})] / \Sigma [(x_i - \bar{x})^2]$$

$$b_1 = 470/730$$

$$b_1 = 0.644$$

Once we know the value of the regression coefficient (b_1), we can solve for the regression slope (b_0):

$$b_0 = \bar{y} - b_1 * \bar{x}$$

$$b_0 = 77 - (0.644)(78)$$

$$b_0 = 26.768$$

Therefore, the regression equation is: $\hat{y} = 26.768 + 0.644x$.

How to Use the Regression Equation

Once you have the regression equation, using it is a snap. Choose a value for the independent variable (*x*), perform the computation, and you have an estimated value ($\hat{y}$) for the dependent variable.

In our example, the independent variable is the student's score on the aptitude test. The dependent variable is the student's statistics grade. If a student made an 80 on the aptitude test, the estimated statistics grade ($\hat{y}$) would be:

$$\hat{y} = b_0 + b_1x$$

$$\hat{y} = 26.768 + 0.644x = 26.768 + 0.644 * 80$$

$$\hat{y} = 26.768 + 51.52 = 78.288$$

Warning: When you use a regression equation, do not use values for the independent variable that are outside the range of values used to create the equation. That is called **extrapolation**, and it can produce unreasonable estimates.

In this example, the aptitude test scores used to create the regression equation ranged from 60 to 95. Therefore, only use values inside that range to estimate statistics grades. Using values outside that range (less than 60 or greater than 95) is problematic.

How to Find the Coefficient of Determination

Whenever you use a regression equation, you should ask how well the equation fits the data. One way to assess fit is to check the coefficient of determination, which can be computed from the following formula.

$$R^2 = \{ (1 / N) * \Sigma [(x_i - \bar{x}) * (y_i - \bar{y})] / (\sigma_x * \sigma_y) \}^2$$

where N is the number of observations used to fit the model, Σ is the summation symbol, x_i is the x value for observation i, $\bar{x}$ is the mean x value, y_i is the y value for observation i, $\bar{y}$ is the mean y value, σ_x is the standard deviation of x, and σ_y is the standard deviation of y.

Computations for the sample problem of this lesson are shown below. We begin by computing the standard deviation of x (σ_x):

$$\sigma_x = \text{sqrt} [\Sigma (x_i - \bar{x})^2 / N]$$

$$\sigma_x = \text{sqrt}(730/5) = \text{sqrt}(146) = 12.083$$

Next, we find the standard deviation of y, (σ_y):

$$\sigma_y = \text{sqrt} [\Sigma (y_i - \bar{y})^2 / N]$$

$$\sigma_y = \text{sqrt}(630/5) = \text{sqrt}(126) = 11.225$$

And finally, we compute the coefficient of determination (R^2):

$$R^2 = \{ (1 / N) * \Sigma [(x_i - \bar{x}) * (y_i - \bar{y})] / (\sigma_x * \sigma_y) \}^2$$

$$R^2 = [(1/5) * 470 / (12.083 * 11.225)]^2$$

$$R^2 = (94 / 135.632)^2 = (0.693)^2 = 0.48$$

A coefficient of determination equal to 0.48 indicates that about 48% of the variation in statistics grades (the dependent variable) can be explained by the relationship to math aptitude scores (the independent variable). This would be considered a good fit to the data, in the sense that it would substantially improve an educator's ability to predict student performance in statistics class.